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AMERICAN JOURNAL of PHYSICS

A Journal Devoted to the Instructional and Cultural Aspects of Physical Science

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Dimensional Theory: Dimensionless Secondary Quantities

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(Received November 19, 1952)

In realms where natural constants, l_0, t_0, or v_0, of length, time, and/or velocity, are present, dimensionless secondary quantities can be defined of the form $Q'=f(l_0/l, t_0/t, v_0/v$, etc.) which are not, in general, expressible as a constant $\times$ the product of powers of primary quantities. It is suggested that dimensionless quantities of the form Q' are required in the theory of interactions.

THIS paper calls attention to an unnecessary restrictive assumption implicit, but inadequately discussed, in the standard treatments of dimensional theory in order to clear the way for a method to be used later.

The assumption is that the only secondary quantities that need be considered are *dimensional*, i.e., represented by numbers which vary with the units of the primary quantities. This assumption is implicit in the derivations of the product of powers theorem, which states that if the ratio Q_1/Q_2 of two quantities of the same kind is to be independent of the primary units (Bridgman's "absolute significance of relative magnitude"), Q must be expressible in the form $Q=cl^\alpha t^\beta m^\gamma$, where c is a constant; l, t, and m are measured variables; and α, β, γ are constants representing the dimensions of Q in the units of length, time, and mass, respectively. The dimensional formula for Q is then $[Q]=[L]^\alpha[T]^\beta[M]^\gamma$ in the usual notation, and α, β, γ are normally constants characteristic of the kind of quantity, though in rheological expressions they may be material constants.[1,2]

But this assumption does not express a necessary requirement of physical theory, and the theorem is invalid if *dimensionless secondary quantities* of a relatively novel kind are admitted. In any realm where natural constants (l_0, t_0, v_0), either universal or material, of length, time, and/or velocity are present it is permissible, and may be advantageous, to define a dimensionless secondary quantity Q' by putting $Q'=f(l_0/l, v_0/v)$ say, where f is any selected function. Quantities Q'_1, Q'_2, and their ratio Q'_1/Q'_2 are then independent of the primary units, the dimensional formula for Q' being $[Q']=[1]$. Since f is arbitrary, Q' is more general than Q, to which it becomes equivalent in special cases, i.e., when f is a product of powers and l_0, v_0, etc., combine to form a single dimensional constant.

Thus the assumption is both unnecessary and restrictive. If the product of powers form appears to be unsuited to any physical problem (a possible example is rheology where traditional methods are inadequate), no dimensional difficulties can arise in seeking to apply the more general form Q'.

Bridgman's conclusion[3] that "every secondary

[1] M. D. Hersey, J. Rheol. **3**, 23 (1932).
[2] G. W. Scott-Blair, Proc. Roy. Soc. (London) **A189**, 69 (1947); H. Dingle, Phil. Mag. **40**, 94 (1949).
[3] P. W. Bridgman, *Dimensional Analysis* (Yale University Press, New Haven 1931), p. 22.

quantity, therefore, which satisfies the requirement of the absolute significance of relative magnitude must be expressible as some constant multiplied by arbitrary powers of the primary quantities" is therefore only valid if dimensionless quantities are excluded by the insertion of the word "dimensional" after "every." This exclusion is implicit in Bridgman's notation, but was not discussed; as there was then little reason to suspect that there existed realms in which all measurements of length (say) might have to be regarded as the determination of the ratio of the measured length to a natural unit of length, as is implied in the use of Q'. A similar comment applies to other more recent references[4,5] to the product of powers theorem. Buckingham's treatment[6] explicitly includes dimensionless quantities representing the ratio of two-dimensional quantities of the same kind, but implies that these are ratios of variables (e.g., two lengths characteristic of a system) and does not consider the possibility of dimensionless quantities of the form Q'.

Dimensionless variables representing primary variables measured in natural units are widely used, e. g., as the scale factor (l_0/l) in Yukawa's potential and in other quantum expressions rendering explicit the presence of a fundamental length; as the time factor (t_0/t) in decay and relaxation processes, rheological expressions, etc.; and as the velocity factor, (v_0/v) or (v/c), in relativistic corrections and particle interactions. But these dimensionless variables have been introduced piecemeal, without their being used to define dimensionless secondary quantities or it being noted that their consistent exploitation in laws of the very general form $F(l_0/l,\ v_0/v,$ etc.,$)=0$ releases physical theory from the traditional restriction to secondary quantities expressible as products of powers of the primary quantities. Bridgman's statement[4] in his valuable reformulation of the ideas of his book, that "no secondary quantities are in present scientific use . . . whose dimensional formulas are not products of powers" is correct in a formal sense, since the dimensional formula of a dimensionless quantity can be obtained by putting $\alpha=\beta=\gamma=0$ in the product of powers expression, but it has the effect of excluding from consideration dimensionless quantities such as Q'.

The catch lies in the fact that while the dimensional formula of a dimensional quantity is, apart from a constant, identical in form with the expression for the quantity in terms of primary measures, the formula for a dimensionless quantity Q', $[Q']=[1]$, does not reveal either the possible presence or the character of the function f. In view of this distinction between the definition and the dimensional formula of a dimensionless quantity part of the literature of dimensional theory becomes ambiguous once the possibility of using dimensionless quantities of the form Q' has been brought into discussion.

The issue raised here may prove to be far from trivial. Dimensionless variables best represent the significant structure of many phenomena, and there is reason to believe that certain unsolved problems require exceptionally powerful methods such as they provide. Idealized processes which are represented *separately* by the use of dimensional constants of the traditional type are characterized by spatial and temporal similarity properties (i.e., by dimensionless products capable of being held constant while the linear or temporal scale of the process is changed),[7] the relevant dimensionless products containing no constant of length or time. But the representation of actual processes involving their *interactions* demands nonlinear terms (or the equivalent) and the presence of a constant of length or time, which implies the impossibility of constructing an exact model with the same constituents, since (l_0/l) cannot be held constant while l is changed. For example, the universal length e^2/mc^2 enters where both light and electrons are present, and a time period characteristic of the state of the material equal to viscosity/elasticity where both viscosity and elasticity are involved. This suggests that a theory of the processes now described as "interactions," whether of fundamental particles or of macroscopic phenomena such as elasticity and viscosity, may require the

[4] P. W. Bridgman, "Dimensional Analysis," *Encyclopaedia Britannica*, 1947 and later editions.
[5] E. W. H. Selwyn, Brit. J. Appl. Phys. **3**, 209 (1952).
[6] E. Buckingham, Phys. Rev. **4**, 345 (1914).

[7] See reference 6 or H. L. Langhaar, *Dimensional Analysis and Theory of Models* (John Wiley and Sons, Inc., New York, 1951), p. 64.

use of functions of dimensionless variables of the form Q'.

Various physical interpretations have been given[8] of certain dimensionless products of the Buckingham II theorem (Reynolds, Prandtl, Peclet, and Nusselt numbers) which represent *ratios of dynamical quantities* characteristic of the system as a whole and determining the local ratios of heat transfers and/or momentum exchanges of different kinds. The detailed consideration of the physical or geometrical significance of the kind of dimensionless quantities discussed here, which are *functions of* simple *kinematical ratios* such as l_0/l and v_0/v, lies outside the scope of this paper, but one example will be given to illustrate the present argument.

In some cases it may be mathematically convenient to use a variable of type Q' defined as an *angle*, by putting some trigonometric function of $\theta=l_0/l$ or v_0/v, such as $\sin\theta=v/c$, in order to benefit by the methods of trigonometry. Moreover, this mathematical transformation is appropriate, since (a) angles are normally measured as secondary quantities, i.e., as ratios of lengths; and (b) it may suggest a physical identification, the θ's being recognized as three-dimensional *structural angles*.[9] Thus the apparent angular deformation ("due to the Fitzgerald-Lorentz contraction"), in a plane passing through v, of the molecular structure of a rigid body, whose velocity is v relative to the observer, is a simple function of θ, where $\theta=\sin^{-1}/(v/c)$, as is well known.

ACKNOWLEDGMENT

I wish to thank Professor P. W. Bridgman and Dr. W. Squire for helpful correspondence.

[8] For example, E. R. Van Driest. J. Appl. Mech. **13**, A34 (1946).

[9] On the role of structural angles see L. L. Whyte, Am. Math. Monthly, **59**, 606 (1952); Brit. J. Phil. Sci. **3**, 256 (1952).

A Course in Radioactive Determinations

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(Received October 15, 1952)

A lecture-laboratory course in the use of radioactive isotopes is discussed. The scope of the material presented should furnish adequate initial training for persons whose future may be expected to include work with radioactive materials.

SINCE 1946, when radioactive isotopes became readily available to laboratories equipped to handle them, it has become increasingly necessary for educational institutions to provide means of training persons in the skills necessary for tracer work. While the Oak Ridge Institute of Nuclear Studies organized a training course in 1947 designed to instruct representatives of groups planning to use isotopes, this training, in general, was inaccessible to students at the college level. Also, while it is practical to send a representative of the college to acquire those skills, it borders on the impossible to send all the staffs of the different departments which can actively benefit from this training.

Because of this, Kansas State College took early steps to provide such training on the campus. A series of lectures covering various phases of radioactivity were given in 1948, and as soon as laboratory facilities were provided, a formal course was started in the fall semester of 1950.

Many reasons can be advanced for giving this type of training, particularly as it affects students. Many government laboratories have a need for persons trained in biology, chemistry, engineering, medicine, and physics who have some knowledge of the additional skills needed in tracer work. Also, not to be ignored is the cultural value it holds in this atomic age for those persons whose employment lies in non-science fields.

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Although the School of Arts and Sciences is the largest on the Kansas State campus, many of the curricula are planned on an applied basis. One finds the curricula of Industrial Chemistry and Industrial Physics the most popular offerings of these respective departments. This in itself heightened the need for a course available to students in these departments to prepare them in the skills necessary for the application to research of nuclear chemistry and physics. Lastly, tracer techniques applied to biology have been described as being the greatest advances in measuring devices since the discovery of the microscope. Because of their special interest in biological and agricultural sciences it is natural that training in this field is popular at land-grant colleges.

PROBLEMS OF ORGANIZATION

Probably no course presented at the present time to college groups requires as much organization as does one in which radioactive materials are to be handled. Since the largest supplier of these materials is the Atomic Energy Commission, its requirements must be the first ones met. Because these requirements change from time to time, no discussion of them will be made here. However, they involve setting up an arrangement by which the personnel involved in the project as well as the general public interests may be protected. At institutions that have cooperative arrangements with National Laboratories, this is accomplished by organizing campus-wide isotope control committees; in some cases central isotope laboratories have been established.

In addition to the administrative problems, the actual equipping of the necessary laboratories requires thought. In our case the laboratories initially were planned to aid our research. In order to expand these facilities to include student instruction, additional equipment was necessary. Such equipment was chosen with the objectives of acquainting the students with different makes and types of equipment and at the same time adding to the versatility of our research facilities.

Combining the functions of research and teaching into one laboratory was at first considered undesirable. Actually no serious problems have occurred other than local conflicting demands for space. Obvious advantages to the student have been the possibility and even the necessity of utilizing much of the latest equipment available for research. This leads to increased experience as well as to better understanding of feasible research problems. On the other hand, careful supervision makes possible student usage of the laboratories with minimum danger of contamination or harm to the equipment. This full utilization tends to keep the apparatus constantly employed and consequently in a better state of repair than would be possible otherwise.

NATURE OF THE COURSE

Students normally enrolled in this course are seniors and first-year graduate students. Their backgrounds may be that of any curriculum of any department on the campus. Thus, it immediately becomes necessary to present a course which is basic in the topics discussed and which, also, is sufficiently advanced to add to the information of a student even in his major field. It is not as difficult to do this as one might imagine. The three classifications of physics, chemistry, and biology provide natural divisions of the subject matter. Our normal student has a major in one of these fields and a minor in one of the others. It is quite possible to discuss details necessary to review phases of each of these fields so as to provide new information for the noninformed student and advanced information to the student majoring in the specific field. We have yet to experience the criticism that the material presented has been a repetition of that offered elsewhere.

The class time devoted to our course includes two one-hour lecture periods and one three-hour laboratory period each week for one semester. This time is augmented by problems, library assignments from time to time, and by occasional extra time spent in the laboratory in measuring activities, i.e., half-life determinations and developing autographs. In this type of experiment, one cannot always arrange for the measurements to be made only at weekly intervals. However, extra laboratory work is kept to a minimum. It is not required that an experiment be completed in a single laboratory period. In fact,

some of the experiments are of such a nature that work is being performed on more than one of them at the same time.

The textbook assigned in this course at the present time was written by Martin D. Kamen and is entitled *Radioactive Tracers in Biology*. However, as might well be expected, no one book covers adequately all of the topics necessary for this rapidly expanding field. An assigned textbook is used mostly as a readily available reference; and the material presented is taken from mimeographed notes given to the students, from reference books of which more will be said later, and from current literature. When possible, lectures are presented to the students by persons doing research in specific fields. This last teaching aid is made possible, or at least the possibility is greatly improved, by the necessary centralization of this type of research. It should be emphasized that these later research talks are given only after the basic procedures have been discussed in a lecture discussion series. Such a procedure makes it possible for the guest lecturer to present his material clearly and quickly with the least confusion due to terminology difficulties.

The mathematical requirements are, of necessity, low. Derivations in which calculus is needed such as those for activities, half-life, and half-thickness are given. The results are considered more important than the derivation, however, and are stressed in the problems which cover this phase of work.

A brief topic outline of the material presented in the lecture-discussion sessions of our course is as follows:

I. Introduction of Nuclear Physics

(A) Introduction: Brief discussion of work of Roentgen, Becquerel, Schmidt, Mme. Curie, Meyer, Von Schweidler, Rutherford, Paneth, and others.

(B) High points of atomic and nuclear theory: Bohr's concept of the atom; energy levels; origin of light and x-rays; quantum numbers; spin of the nucleus; elementary particles; bombardment of the nucleus; nuclear well; Coulomb forces; possible exchange forces; equivalence of mass and energy; packing fraction; fission and fusion processes.

(C) The laws of radioactive decay: Activity of a sample; half-life; determination of decay constant; parent and daughter relationships; curie; rutherford.

(D) Nuclear reactions and modes of decay: Particles and energies necessary for specific transmutations; isomeric transitions; decay schemes; beta-decay; neutrino; allowed and forbidden transitions.

(E) Interaction of radiation with matter: Charged and uncharged particle scattering; bremsstrahlung; Compton effect; photoelectric effect; pair production; atomic and molecular effects; roentgen unit; roentgen-equivalent-physical, roentgen-equivalent mammal; *n* unit; energy measurements through absorption; Feather analysis; half-thickness; cross section.

(F) Instrumentation and procedures of activity measurements: Emulsions; scintillations; electrometers; ionization chambers; Geiger-Mueller tubes; scalers; proportional counting; coincidences; dead time; backscattering; sample preparation; sample absorption; counting standards; counting statistics.

(G) Tracer isotope production: Simplified reactor theory; accelerators; fission products; uranium reactor-produced activities.

(H) Health physics: Tolerance levels; personnel protection; radiation damage; laboratory design; survey meters; dosimeters.

II. Radiochemical Aspects

(A) Significance of tracer methods: Sensitivity of method; mass effect of isotopes on chemical reaction; dilution factors which are possible compared to stable isotopes.

(B) Desirable characteristics of a tracer as applied to any particular problem: Parent-daughter and isomer effects; type of radiation; ionizing effect; some aspects of radiation chemistry.

(C) "Hot-atom" chemistry and related topics: Szilard-Chalmers effect and criteria for success of such a reaction.

(D) Proper labeling of a molecule of complex ion: Relation of exchange to bond types and chemical compounds; exchange studies between ions, between ions and molecules, between ions and solids.

(E) Principles of tracer chemistry: Types of carriers; co-precipitation studies; fission product chemistry; resin and chromatographic techniques for tracer amount of materials.

(F) Synthesis methods for labeling a specific atom in a molecule: Heterogeneous labeling of high molecular weight compounds such as proteins; labeling of hormones, fatty acids by biosynthetic methods; comparison of chemical and biosynthesis methods.

(G) Radiation effects and inhibition by certain compounds: Deposition of tracers in the body especially C^{14}.

III. A Partial List of Applications

(A) Radiometric analysis applied to insoluable precipitates: Activation analysis for trace quantities; solubility determinations; surface area measurements; isotopic dilution method; self-diffusion studies.

(B) Tracers in geology: Age of earth measurements; C^{14} and H^3 dating.

(C) Metabolism studies: Phosphorus, sulfur, carbon, and iodine compounds; tracers in medical diagnosis and therapeutic treatment.

(D) Tracers in industry: Thickness measurements of films; metallurgical diffusion studies; x-ray studies with Co^{60} and other γ-emitters; uses in the petroleum industry; friction studies; cold sterilization of drugs and foodstuff.

(E) Radioautographic uses: Photographic procedures; microautograms as applied to cell structure and deposition studies.

EXPERIMENTS USED TO EMPHASIZE THEORY

The primary objective of our laboratory work is to acquaint the students with as many different types of apparatus and procedures as is possible. At the same time health physics practices must be constantly brought to the attention of the student.

To facilitate this, it is planned that a new experiment will be started each laboratory period. This plan may limit the degree of accuracy of certain experiments, but is felt necessary in view of the large amount of material which must be covered. In extreme cases, counting of prepared samples may be necessary outside of the regular laboratory periods. To compensate for excessive laboratory time, laboratory report requirements are kept to a minimum. This may be partially justified in terms of the number of departments which are represented in a normal class. It is felt unnecessary and, in fact, undesirable to spend any considerable amount of time in directing the form of report writing. Sufficient details are required in a report, however, to allow for the possibility of its being used as a future reference. The use of our mimeographed laboratory directions as part of the write-up reduces the students' work largely to compiling and interpretating data and answering questions concerning theory.

Our laboratory facilities include a preparation room containing three hoods designed to handle different orders of magnitude of activities, adjoining work areas, and a concrete vault for the storage of activities not being used. Housed in this laboratory are the usual supplies and apparatus including micropipettes, chemicals, stainless steel trays, shields, remote control apparatus, balance, centrifuge, etc. An adjoining counting room houses various types of scalers, counting chambers, an electrometer, and the health physics dosimeters and survey meters.

In order to assure as complete coverage of pertinent material as possible, the following experiments are offered to the students. All of these are not performed each semester depending on the needs of the class. In general, however, because of the organization necessary to produce a specific experiment, all students do the same experiments.

(*1*) *Introduction to the Radioisotopes Laboratory.* Students are informed of the hazards and means of protection that are necessary parts of radioactive tracer work. Lists of laboratory regulations are given students. Arrangements are made for obtaining blood counts. The use of dosimeters and survey meters is demonstrated and practiced.

(*2*) *Study of a Geiger Counter and Scaler.* The threshold and operating voltages of a G-M counter are determined and various scalers employed to make simple measurements of an activity at different shelf heights and for various counting times. Counting yield and statistics are stressed.

(*3*) *Use of a Lauritsen Electrometer in Tracer Applications.* A Lauritsen electrometer, modified, to allow insertion of the sample into the counting chamber and also to allow counting at various shelf heights with the sample outside the chamber is used. Linearity of the electrometer is determined. A low energy beta-emitter is counted inside and outside the chamber to demonstrate window absorption. A half-life count of P^{32} is started to demonstrate an electrometer's stability. Half-life is measured as well with a G-M counter.

(*4*) *Determination of Backscattering and Dead Time with a G-M Counter.* Backing materials of aluminum, glass, copper, platinum (or lead) are used with a P^{32} sample prepared on polystyrene to determine the dependence of backscattering on atomic number. Two separate activities are

counted in the usual manner to determine the G-M tube's dead time.

(*5*) *Determination of Energy by Absorption Methods.* Measurement of energies of emitted radiation as a partial means of identifying radioactive substances is made. Aluminum absorption plots are made for P^{32} and Ra D sources. A Feather analyzer is prepared, and the energy of the radiation from P^{32} is determined. Half-thickness and energy are determined for Na^{22}.

(*6*) *Determination of Ionic Exchange Properties.* Exchange between ionic Ca^{45} and calcium bonded to protein in blood serum is demonstrated and the degree of exchange measured. The ratio of bound to ionic calcium may be determined after equilibrium is attained.

(*7*) *Determination of Exchange Rates and Surface Area.* Exchange rates and completeness of exchange may be determined for radioactive silver ions with silver chloride. A similar experiment with silver metal demonstrates the techniques in determining surface area.

(*8*) *Determination of Solubilities.* Solubilities of such labeled compounds as silver halides or zinc sulfide are determined and checked against solubilities determined by other methods. The rate of solubility is also readily checked by this method.

(*9*) *Quantitative Determination by Radiometric Means.* Analysis of an unknown chloride solution by titration with radioactive silver is performed. Modifications of this experiment using other elements may be performed.

(*10*) *Determination of Volume by Isotopic Dilution.* Volume of a system is determined by measuring its specific activity after a known activity has been mixed into it. This may be carried out on an inorganic system, but is more often performed in measuring the blood volume of a rat.

(*11*) *Determination of Separation by Tracer Means.* Compounds are separated by use of resin columns and paper chromatography, and the completeness of separation is measured by means of tracers incorporated into the compounds being separated.

(*12*) *Determination of Metabolism and Deposition of P^{32} in a Biological System.* Deposition of P^{32} after injection in the principal organs of a young chicken or rat is investigated. Time may be incorporated into the experiment by sacrificing the animals at spaced intervals of time.

(*13*) *Determination of Deposition by Autographic Techniques.* Deposition of P^{32} in plant and/or animal tissue is determined by using photographic techniques. The experiment may be made more complete by allowing different student groups to use different types of photographic materials—x-ray no screen, lantern slide, stripping film, liquid emulsion, etc.

Contamination determination and elimination as well as "hot shipment" handling are discussed and demonstrated as the need arises during the course of the semester.

TEXTBOOKS USEFUL AS REFERENCES

A complete listing of reference material for this course of necessity would include many books in the fields of biology, chemistry, and physics. It would include, also, much of the published literature and certainly the printed materials available on request from the AEC.

The following listing is only of those books which we have found especially useful in our work and should by no means be considered as complete.

G. Freidlander and J. W. Kennedy, *Introduction to Radiochemistry* (John Wiley and Sons, Inc., New York, 1949). As the basic text in the field of radiochemistry, this book provides much of the background material for the course. New applications in the field of chemistry and biological sciences must be taken from other sources however.

Martin D. Kamen, *Radioactive Tracers in Biology* (Academic Press Inc., New York, 1951). This is probably the text which most nearly parallels our course. Its discussion of the biologically important elements is excellent. Material is added, however, in the nuclear physics, chemistry, autographic, tracer production, and health physics sections.

R. E. Lapp and H. L. Andrews, *Nuclear Radiation Physics* (Prentice-Hall, Inc., New York, 1948). This is one of the textbooks written shortly after the war which discusses the background material of nuclear physics. The level of the text is such as to not discourage nonphysics students and is in addition stimulating to advanced students of any level. Of particular interest is the health physics section which is omitted in many nuclear physics texts.

George K. Schweitzer and Ira B. Whitney, *Radioactive Tracer Techniques* (D. Van Nostrand Company, Inc., New York, 1949). This is an excellent manual for an elementary radiochemistry course. It presupposes a knowl-

edge of nuclear processes or assumes that such a knowledge is not necessary for elementary tracer work. Its list of experiments is well chosen and is probably the most valuable portion of the book.

William E. Siri, *Isotopic Tracers and Nuclear Radiation* (McGraw-Hill Book Company, Inc., New York, 1949). This is an excellent reference of an advanced nature, and is the most complete of texts listed.

Arthur C. Wahl and Norman A. Bonner, *Radioactivity Applied to Chemistry* (John Wiley and Sons, Inc., New York, 1951). This condensation of a literature search is most useful in making the chemistry student aware of the possibilities of the use of tracers. The book has been written at the senior or first-year graduate student level, and emphasizes the chemical rather than the nuclear aspects of the subject. When used in conjunction with the Friedlander and Kennedy book, it offers a wealth of material for a radiochemistry course or courses.

The Physics of Microwave Calorimeters

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(Received October 24, 1952)

The electromagnetic reflection characteristics of a system of nonideal isotropic dielectrics in nearly ideal wave guides are formally developed. The general analytical results are applied to several cases of practical importance. Consideration is given to the manner of deriving optimum characteristics of flow calorimeters normally used for microwave measurements of power. Other applications include the determination of the complex dielectric constant of high loss liquid dielectrics. Considerations relating to bandwidth of microwave calorimeters are included.

I. PREFACE

THE electromagnetic reflection characteristics of a system of isotropic nonideal dielectrics in a wave guide with normal metallic termination will be formally developed. These results are applicable for dominant *transverse electric* and *transverse magnetic* modes of propagation. Although the attenuation in the waveguide walls may be added to that of the enclosed dielectric, such a correction is generally not important in practice and may be neglected.

Applications include the determination of the reflection characteristics of single and compound windows which confine a dissipative liquid such as water in a shorted circular wave guide. Such an arrangement is used to measure power from magnetrons, klystrons, etc., by flow calorimetric methods.[1] The reflection characteristics of such a calorimeter over the desired frequency band should generally correspond to matched line conditions, especially when the power level approaches the capacity of the wave guide itself. To increase the bandwidth beyond that obtainable from the single disk window, compound windows may be resorted to. The necessary relations from which the desired bandwidth characteristics may be determined for single and compound windows are included in this article.

A circular wave-guide calorimeter with disk type window, as illustrated by Fig. 1 in principle, is especially well suited for high power operation. This inherent advantage is due to the lack of the usual protuberances[2] in the wave guide. Although the thermal capacity of the calorimeter is controlled primarily by the frequency and power level in that these factors control the diameter and length, it is tolerable for most applications at frequencies in excess of about 3000 Mc. To decrease the length of the water column and thereby reduce the thermal capacity, certain electrolytes may be added to the water so that the dissipation factor is increased. However, this alternative requires use of a recirculating system and has not been popular even for aqueous dehydroxy alcohols.

The conventional single disk window (coaxial[3] or wave-guide) type of calorimeter, such as illustrated in principle by Fig. 1, was evolved on the basis of the quarter-wave plate analogy in optics.[4] The primary assumptions involve ideal media of infinite extent on either side of the

[1] Montgomery, *Techniques of Microwave Measurements* (McGraw-Hill Book Company, Inc., New York, 1947), Vol. 11, Sec. 3.35.

[2] Reference 1, Sec. 3.34.

[3] Reference 1, Sec. 3.33; U. S. Patent 2,262,134.

[4] Stratton, *Electromagnetic Theory* (McGraw-Hill Book Company, Inc., New York, 1947), Sec. 9.11.

dielectric plate. Hence, the water column for a calorimeter must be made long enough so that the residual reflection due to the shorted termination in the wave guide is negligible. However, circumstances arise in practice especially below 3000 Mc wherein it is desirable to limit the length of the water column so that these simplifying assumptions are inadmissible. Therefore, a general theoretical analysis is included for this type of calorimeter. It is shown that when the previous simplifying assumption is introduced into the derived general reflection relations the usual quarter-wave plate characteristics are obtained.

To achieve a substantial increase in the bandwidth of calorimeters beyond that obtainable with a single dielectric window, a compound window may be resorted to. The analytical relations and conditions imposed on such a system in order to realize the desired bandwidth are included in this article.

By direct or graphical solution of the derived analytical relations, which characterize single or compound windows confining dissipative liquids in a shorted wave guide, optimum parameters of the system may be determined in terms of the desired characteristics.

The effect of variation in temperature of the dissipative fluid on the reflection characteristics of the calorimeter may also be determined graphically from the appropriate relations derived in this article.

The mks system of units is used throughout this article.

II. PRELIMINARY CONSIDERATIONS

The dominant electromagnetic wave in a medium j enclosed by a wave guide may be expressed as an exponential function,[5]

$$A_j e^{i2\pi\nu t+x\gamma_j}, \tag{1}$$

in which x denotes the axial coordinate of the system and t denotes the time. The symbols A_j, ν, and γ_j denote, respectively, the amplitude and frequency of the wave and complex propagation factor[5] of the medium j. The complex propagation factor is comprised of a real part, or the attenuation factor, and the imaginary part, or

[5] Reference 4, Chaps. V and IX, esp. Secs. 5.2, 9.4 to 9.13 for fundamentals.

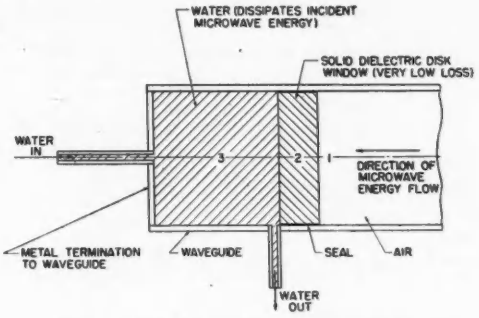

FIG. 1. Principle features of a single window wave-guide calorimeter.

the phase factor or wave number. The propagation factor for the transverse electric, TE, and transverse magnetic, TM, waves may be expressed by the relation[5]

$$\gamma_j^2=(2\pi/\lambda_c)^2+i2\pi\nu\mu_j(i2\pi\nu\epsilon_j+\sigma_j), \tag{2}$$

where the symbols ϵ_j, μ_j, and σ_j denote, respectively, the values of complex dielectric constant, complex permeability, and conductivity of the medium j. The symbol λ_c denotes the cut-off wavelength in the wave guide with the mode of propagation involved.

The longitudinal intrinsic impedance[5,6] is defined as

$$z_j=(\mathbf{n}\times\mathbf{E}_j)/\mathbf{H}_j, \tag{3}$$

for transverse components of electric $\mathbf{E}_j$ and magnetic $\mathbf{H}_j$ fields in the medium j. Here $\mathbf{n}$ denotes the unit vector along the direction of propagation. For the TE wave in an isotropic dielectric this impedance reduces to[5]

$$z_j=i2\pi\nu\mu_j/\gamma_j, \tag{4}$$

and for the TM wave it reduces to[5]

$$z_j=\gamma_j/(i2\pi\nu\epsilon_j+\sigma_j). \tag{5}$$

The complex dielectric constant ϵ_j is comprised of a real part or relative dielectric constant ϵ_j' and an imaginary part or loss factor, ϵ_j'', which are related in the following manner:

$$\epsilon_j=\epsilon_0(\epsilon_j'-i\epsilon_j''), \tag{6}$$

where the symbol ϵ_0 denotes the dielectric constant of free space. The ratio ϵ_j''/ϵ_j' is defined as

[6] Ragan, *Microwave Transmission Circuits* (McGraw-Hill Book Company, Inc., New York, 1948), pp. 52–53.

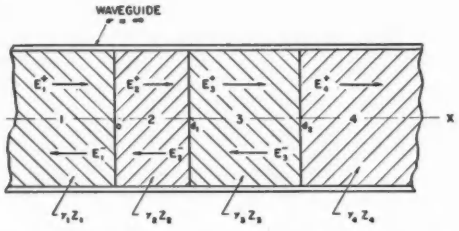

FIG. 2. An ideal uniform wave guide filled with four nonideal isotropic dielectrics separated by plane boundaries.

the loss tangent of the dielectric $\tan\delta_j$. The values of ϵ_j' and $\tan\delta_j$ or power factor $\sin\delta_j$ are usually found in the tables of the physical properties of dielectrics.

III. GENERAL AND PARTICULAR CASES

For the general case of r dielectric plates in the wave guide as illustrated by Fig. 2, the relative amplitude and phase of the transverse electric field of the reflected wave E_1^- to the incident wave E_1^+ at d_j may be expressed in compact two-component matrix notation[7] as

$$\left\|\begin{matrix}E_1^+\\E_1^-\end{matrix}\right\| = \left[\prod_{j=1}^{r-1}\right] \left\|\begin{matrix}e^{i\gamma_j d_j}, & 0\\ 0, & e^{-i\gamma_j d_j}\end{matrix}\right\| \times \tfrac{1}{2}\left\|\begin{matrix}1+n_j/n_{j-1}, & 1-n_j/n_{j-1}\\ 1-n_j/n_{j-1}, & 1+n_j/n_{j-1}\end{matrix}\right\| \left\|\begin{matrix}E_r^+\\E_r^-\end{matrix}\right\|, \quad (7)$$

wherein $n_j = z_0/z_j$ denotes the reduced impedance with respect to the jth medium. To preserve generality, the normal plane in medium 1 of Fig. 2 on the left is considered the reference from which other normal planes of dielectric sections at d_j are measured in the $+x$ direction, toward the right.

Incidentally, a pictorial interpretation of the complex matrix product (7) is provided by the Smith chart[8] which is a transmission line chart based on two sets of orthogonal circles. For example, the ratio E_1^-/E_1^+ corresponds to the radius vector of this chart and the first matrix in the product of expression (7) corresponds to rotation with exponential decay of this ratio.

In order to analyze the important practical case of three dielectrics in a shorted wave guide as illustrated by Fig. 3, it is convenient to reduce the general expression (7) so that it is applicable to four dielectrics, and then let the propagation factor $\gamma_4 \to 0$ and the impedance $z_4 \to 0$ so as to introduce the effect of the shorted termination of the wave guide. However, for instructional purposes it is better to develop the applicable relation formally in the manner to be described presently.

As illustrated by Fig. 2, an ideal uniform wave guide is filled with four general isotropic dielectrics, 1, 2, 3, and 4, of which, 2 and 3, are of finite thickness, d_1 and d_2, whereas, 1 and 4, are unlimited in thickness. The interfaces of the dielectric are perpendicular to the axis of the wave guide, which is also the coordinate x axis.

By virtue of the inhomogeneity of the composite dielectric in the wave guide, electromagnetic waves are established in the four media because of the partial reflections at the interfaces of the dielectrics at d_1 and d_2. These waves are characterized by a set of incident, E_j^+, and reflected, E_j^-, transverse components of electric, $\mathbf{E}_j$, and magnetic, $\mathbf{H}_j$, fields in the four dielectrics as follows

x	TE	TM	
$x \le 0$	$E_1^+ e^{-\gamma_1 x}$	$(E_1^+/z_1)e^{-\gamma_1 x}$,	(8)
	$E_1^- e^{+\gamma_1 x}$	$-(E_1^-/z_1)e^{+\gamma_1 x}$,	(9)
$0 \le x \le d_1$	$E_2^+ e^{-\gamma_2 x}$	$(E_2^+/z_2)e^{-\gamma_2 x}$,	(10)
	$E_2^- e^{+\gamma_2 x}$	$-(E_2^-/z_2)e^{+\gamma_2 x}$,	(11)
$d_1 \le x \le d_2$	$E_3^+ e^{-\gamma_3 x}$	$(E_3^+/z_3)e^{-\gamma_3 x}$,	(12)
	$E_3^- e^{+\gamma_3 x}$	$-(E_3^-/z_3)e^{+\gamma_3 x}$,	(13)
$x \ge d_2$	$E_4^+ e^{-\gamma_4 x}$	$(E_4^+/z_4)e^{-\gamma_4 x}$.	(14)

These transverse components of electric and magnetic fields illustrate the exponential character of the waves in the various media j as a function of the axial distance x in them.

Application of the boundary conditions[5] provides the following six relations between the seven electric magnitudes, $E_1^{\pm}$, $E_2^{\pm}$, $E_3^{\pm}$, and E_4^+ of the field vectors.

At $x=0$, for the TE wave, the transverse fields are related in the following manner;

$$E_1^+ + E_1^- = E_2^+ + E_2^-, \quad (15)$$

and for the TM wave the relationship is

$$E_1^+ - E_1^- = (z_1/z_2)(E_2^+ - E_2^-). \quad (16)$$

[7] E. U. Condon, Revs. Modern Phys. **14**, 341–389 (1942), esp. pp. 383–385, for fundamentals.
[8] Reference 6, Sec. 2.12.

At $x=d_1$, for the TE wave, the transverse components of the electric fields are related in the following manner:

$$E_2^+e^{-\gamma_2 d_1}+E_2^-e^{+\gamma_2 d_1}=E_3^+e^{-\gamma_3 d_1}+E_3^-e^{+\gamma_3 d_1}, \quad (17)$$

and for the TM wave, the relationship is

$$E_2^+e^{-\gamma_2 d_1}-E_2^-e^{+\gamma_2 d_1} =(z_2/z_3)(E_3^+e^{-\gamma_3 d_1}-E_3^-e^{+\gamma_3 d_1}. \quad (18)$$

At $x=d_2$, for the TE wave, the transverse components of the electric fields are related in the following manner

$$E_3^+e^{-\gamma_3 d_2}+E_3^-e^{+\gamma_3 d_2}=E_4^+e^{-\gamma_4 d_2}, \quad (19)$$

and for the TM wave the relationship is

$$E_3^+e^{-\gamma_3 d_2}-E_3^-e^{+\gamma_3 d_2}=(z_3/z_4)E_4^+e^{-\gamma_4 d_2}. \quad (20)$$

To obtain the complex ratio of transverse electric field of the reflected wave to that of the incident wave E_1^-/E_1^+ it is necessary to match the tangential fields at a particular boundary. In other words, it is necessary to equate the transverse field components at $x=0$. Similarly, the corresponding ratio of the transmitted wave at $x=d_2$ to the incident wave at $x=0$, $E_4^+e^{-\gamma_4 d_2}/E_1^+$, can be obtained if desired. The following particular function results after the former operation

$$\frac{E_1^-}{E_1^+}=\frac{Ae^{\theta}+Be^{-\theta}+Ce^{\phi}+De^{-\phi}}{Ee^{\theta}+Fe^{-\theta}+Ge^{\phi}+He^{-\phi}}, \quad (21)$$

where the impedance coefficients are

$$\begin{aligned}
A&=(z_2-z_1)(z_2+z_3)(z_4+z_3),\\
B&=(z_2+z_1)(z_2+z_3)(z_4-z_3),\\
C&=(z_2-z_1)(z_2-z_3)(z_4-z_3),\\
D&=(z_2+z_1)(z_3-z_2)(z_4+z_3),\\
E&=(z_2+z_1)(z_2+z_3)(z_4+z_3),\\
F&=(z_1-z_2)(z_2+z_3)(z_4-z_3),\\
G&=(z_1+z_2)(z_2-z_3)(z_4-z_3),\\
H&=(z_1-z_2)(z_2-z_3)(z_4+z_3),
\end{aligned}$$

and phase angles are

$$\begin{aligned}
\theta&=\gamma_2 d_1-\gamma_3(d_1-d_2),\\
\phi&=\gamma_2 d_1+\gamma_3(d_1-d_2).
\end{aligned}$$

The particular function (21) is applicable to any combination of four or fewer arbitrary isotropic dielectrics. It represents the relative amplitude and phase of the transverse electric field of the reflected and incident wave at $x=0$. The amplitude and phase factors can be determined in terms of the intrinsic impedances z_j, propagation factors γ_j, and thickness d_j of the component dielectrics of whatever conductivity σ_j.

One effective manner of portraying the characteristics of the wave guide filled with four or fewer dielectrics is to plot as ordinates the phase and magnitude of the ratio E_1^-/E_1^+ as functions of d_1/λ or d_2/λ with either d_2/λ or d_1/λ, respectively, as a parameter. The symbol λ denotes the free-space wavelength.

With three media, for which $d_2=d_1=d$, $\gamma_4=\gamma_3$, and $z_4=z_3$, the function (21) reduces to the simpler expression

$$\frac{E_1^-}{E_1^+}=\frac{(z_2-z_1)(z_3+z_2)e^{\gamma_2 d}+(z_2+z_1)(z_3-z_2)e^{-\gamma_2 d}}{(z_2+z_1)(z_3+z_2)e^{\gamma_2 d}+(z_2-z_1)(z_3-z_2)e^{-\gamma_2 d}}, \quad (22)$$

and becomes even simpler for lesser number of dielectrics. Incidentally, both functions (21) and (22) are directly applicable to the study of reflections from coated windows[4] when media 1 and 4 in Fig. 2 are identical, for example, air. For the latter case, $\gamma_1=\gamma_4=\gamma$ and $z_1=z_4=z$. This case will be considered in the following section.

IV. PRACTICAL APPLICATIONS

A combination of considerable importance in microwave applications comprises air dielectric for medium 1, a relatively low loss dielectric disk window for medium 2, a relatively high loss tangent liquid dielectric for medium 3 and a highly conducting metal for medium 4 as illustrated by Figs. 1 and 3. The latter restriction requires that $\gamma_4\to\infty$ and $z_4\to 0$. Hence, function (21) becomes

$$\frac{E_1^-}{E_1^+}=\frac{Ie^{\theta}+Je^{-\theta}-Ke^{\phi}+Ie^{-\phi}}{-Je^{\theta}+Ie^{-\theta}+Le^{\phi}-Ke^{-\phi}}, \quad (23)$$

where the impedance coefficients are

$$\begin{aligned}
I&=(z_1-z_2)(z_2+z_3),\\
J&=(z_1+z_2)(z_2+z_3),\\
K&=(z_1-z_2)(z_2-z_3),\\
L&=(z_1+z_2)(z_2-z_3).
\end{aligned}$$

By means of relation (23) the relative amplitude and phase of the transverse electric field of the reflected and incident waves E_1^- and E_1^+ at $x=0$ may be evaluated for the case represented by Fig. 3.

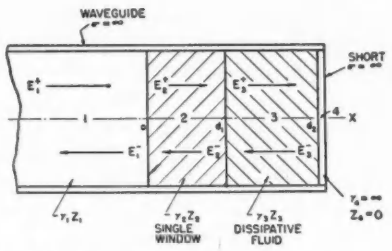

FIG. 3. An ideal uniform shorted wave guide filled with two nonideal isotropic dielectrics, one a dissipative fluid.

The necessary condition that the ratio of the transverse electric fields E_1^-/E_1^+ be zero at $x=0$ is that the numerator of function (23) shall vanish. That is

$$Ie^{\theta}+Je^{-\theta}-ke^{\phi}+Ie^{-\phi}=0. \qquad (24)$$

The condition represented by relation (24) may be difficult to fulfill, in general, so that it may be necessary to return to function (23) and apply a minimum theorem[9] to the function of the two variables d_1/λ and d_2/λ. An alternative procedure is to make either d_1/λ or d_2/λ a parameter and apply the usual minimum theorem[9] or solve graphically. Practically, since medium 3 is generally a liquid dielectric and medium 2 a solid dielectric, d_1/λ is selected as parameter.

It is judicious in practice to reduce the amount of labor indicated by function (23) to a minimum. This may be done by applying several valid approximations without materially affecting the critical properties of function (23). Certainly, for air as medium 1 in Fig. 3 γ_1 may be assumed to be imaginary and z_1 to be real. Furthermore, the window as medium 2 is chosen so that γ_2 is to a high order approximation imaginary and z_2 real. This expediency is necessary for several reasons, of which one is to reduce thermal stresses in the window. For such working fluids as water for medium 3, γ_3 and z_3 are generally complex.

Condition (24) can be further simplified when E_3^- is negligible and all the dielectrics in the system of Fig. 3 are nearly ideal. For this case, E_1^-/E_2^+ in relation (22) vanishes, if

$$(z_2-z_1)(z_3+z_2)e^{\gamma_2 d}+(z_2+z_1)(z_3-z_2)e^{-\gamma_2 d}=0. \qquad (25)$$

This condition holds only when $d=n\lambda_2/4$, where n is an odd integer and z_2 is the geometric mean of z_1 and z_3. That is $z_2^2=z_1z_3$. The first requirement defines the quarter-wave plate analogy in optics previously referred to. When these relations are employed as first-order approximations in the design of single disk window coaxial line calorimeters,[3] laboratory measurements are applied to determine the degree of correction required to compensate for the observed deviations in the desired characteristics.

For calorimetric application at frequencies below 3000 Mc, the water column lengths must be made so short in order to realize a tolerable thermal capacity that the first-order approximations just cited are inadmissable. Under such circumstances there is no other choice but to use the more general analytical relations like Eq. (24) with the less restricting simplifications cited earlier or else resort to a laborious cut-and-try method of approach to the optimum parameters by graphical processes.

When a compound window is needed to achieve greater bandwidth than is realizable with the single window calorimeter, as illustrated by Figs. 1 and 3, the labor required to approach optimum parameters by impedance measurements becomes manyfold greater. In every case, the solution of the particular analytical relations yields more information and the optimum parameters are more conveniently presented and probably with much less expense and effort than would be required by such laboratory methods. In any case, however, the complex dielectric constants of the dielectric components of the calorimeter must be known[10] or measured within the frequency and temperature ranges of interest.

Incidentally, the real and reactive components

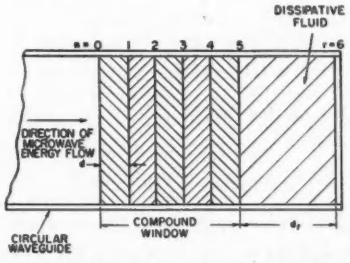

FIG. 4. An ideal uniform shorted wave guide filled with five solid low loss isotropic dielectrics and a dissipative fluid.

[9] Chaundy, *The Differential Calculus* (Oxford University Press, London, 1935), Chap. IX, Sec. 6.

[10] NDRC, MIT-RL Reports 14–237, 14–300, 14–425 (Library of Congress, Reference Department, Science Div., Navy Research Sec., Washington D. C., 1944–1945).

of the impedance z at $x=0$ may be evaluated from any of the derived complex reflection coefficients $|E_1^-/E_1^+|^2$ in the usual manner. Whenever the maximum bandwidth[11] of a microwave calorimeter is of paramount consideration, optimum ratios between successive characteristic impedance steps of the nearly ideal dielectric sections of the compound window are sought. In general, to determine these the thicknesses or axial lengths of the successive sections may be made parameters and the amplitude of the reflected to incident waves or over-all reflection ratio $|E_1^-/E_1^+|$ at $x=0$ solved as previously indicated.[12]

This problem for $n=3$ steps is most expediently resolved by substitution of the most probable characteristic impedance group into the over-all reflection relation appropriate to the number of sections, or constructed in the manner to be described presently and solved for the resultant reflection at $x=0$ as a function of the length of the dissipative fluid column d_r in Fig. 4.

An attractive impedance group[13] derived on the bases of ideal dielectrics and negligible waveguide termination reflection, is characterized by the property that each successive characteristic impedance step in the group acts as a quarter-wave transformer[13] at mid-band. That is, the discontinuities in the logarithms of the characteristic impedance steps of order $(r-1)$ are in proportion to the successive sets of binomial coefficients. Another attractive impedance group for this purpose is characterized by the Tschebyscheff coefficients.[14] Although the foregoing has been derived on the bases of ideal dielectrics in order to portray simply the essential principles, the actual or dissipation case can be resolved in a manner formally identical with it. It is necessary only in the dissipative case to use the complex propagation constant.

A dielectric mixture can provide the characteristic impedance required in making wide band windows for wave-guide calorimeters. One satisfactory mixture[15] is available commercially for a single window in a wave-guide calorimetric application as illustrated by Figs. 1 or 3. For broad band compound windows a rutile[16] binary mixture is applicable. In this case each section of the compound window may be combined in terms of volume percentages v with other ceramics[10] or thermoplastic synthetic resins,[10] according to a logarithmic mixture formula

$$\ln\epsilon=v_1\ln\epsilon_1+v_2\ln\epsilon_2, \tag{26}$$

due to Licktenecker and Rother.[16] The range of dielectric constants which can be realized with such low loss binary mixtures is quite adequate for most applications.

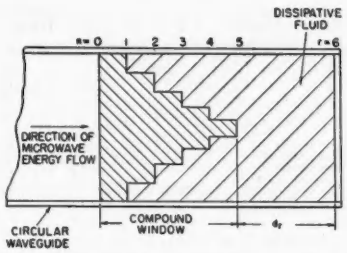

FIG. 5. An ideal uniform shorted wave guide filled with a binomial-stepped solid low loss dielectric and a dissipative fluid.

As an alternative procedure, a solid dielectric binomial matching window can be resorted to as illustrated by Fig. 5. Such a matching device is practical to construct from a solid low loss dielectric whose real dielectric constant is negligible compared to that of the fluid. To determine the radii to the successive interfaces between the fluid and solid dielectric of each section requires a solution of the field equations in a circular wave guide with two coaxial dielectrics for the particular mode of propagation considered.[17] An alternative procedure involves the application of the radial transmission line theory[18] on an impedance basis. That is, the characteristics of the dominant mode in the circular wave guide with two coaxial dielectrics may be resolved by regarding the transmission along the transverse directions perpendicular to

[11] Fano, J. Franklin Inst. **249**, 57–83 and 139–154 (1950).
[12] Reference 9, Sec. 8.
[13] Selgin, *Electrical Transmission in the Steady State* (McGraw-Hill Book Company, Inc., New York, 1946), Secs. 10.2 and 10.3.
[14] Reference 6, p. 591.
[15] A low loss glass-mica mixture is manufactured and *machined to specifications* under the trade name of "Mycalex 400 and 401" by the Mycalex Corporation of America, 60 Clifton Blvd., Clifton, New Jersey. This dielectric may be sealed to the wave guide by Apiezon Wax No. 4 distributed by the James G. Biddle and Company, 1316 Arch Street, Philadelphia 7, Pennsylvania.
[16] Berberich and Bell, J. Appl. Phys. **11**, 681–692 (1940).
[17] Frankel, J. Appl. Phys. **18**, 650 (1947).
[18] Montgomery, Dicke, and Purcell, *Principles of Microwave Circuits* (McGraw-Hill Book Company, Inc., New York, 1948), Chap. 8.

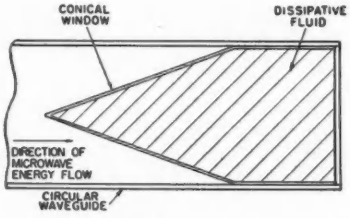

FIG. 6. An ideal uniform shorted circular wave guide with a conical taper window and a dissipative fluid.

the wave-guide axis. This transverse network is equivalent to the original wave guide.

For high power applications in which the thermal capacity of the fluid in the calorimeter is of negligible concern compared to the ease with which a required wide bandwidth characteristic may be obtained, it is simpler to resort to linear conical fluid in circular wave guide. This type of calorimeter comprises a thin wall low loss dielectric cone with its axis coaxial with that of the wave guide as illustrated by Fig. 6. The fluid in the cone is almost invariably tap water in a non-recirculating type of system. The design of such dielectric tapers may be readily resolved, for ideal dielectrics, in a number of ways.[19] The results for dissipative dielectrics may be obtained in a manner formally identical with the ideal case; in the dissipative case the complex rather than the real propagation constants must be used. It is usually more expedient to avoid this complication by using the results for the ideal case as an approximate solution and applying the necessary corrections by low level impedance measurements.[20] Unusual care must be observed in the design and fabrication of conical taper loads so as to avoid hot spots, especially at the apex of the cone in Fig. 6, and materials for the window should be chosen which have low dielectric loss and do not carbonize if arcing should occur. The base of the cone should fit snugly inside the wave guide, and the gaps remaining should be sealed with a low loss high dielectric strength sealing compound, for example, Dow Corning's ignition sealing compound No. 4. The power capacity and strength of the cone loads are limited by the protuberance and volume of fluid required.

There are many important technological details associated with the design and construction of reliable microwave calorimeters which obviously cannot be given adequate treatment in this article. However, the following few comments are permissible:

(1) The electric gradient in the water should be kept below values which affect its dielectric constant and strength.[21]

(2) A small continuous metal indentation or ledge on the wave-guide wall in a plane normal to the axis of the wave guide is highly recommended for supporting the window at its rim. Thereby the water pressure against the window will not be transferred to the wax seal as a shearing force. It is important, at high power, to avoid open blisters in the wax seal where they may be exposed to the air side of the window adjacent to the ledge. If some blisters occur in spite of precautions, smear Dow Corning's ignition sealing compound No. 4 into pits and crevices in the exposed wax and wipe off the excess.

(3) For a narrow band single window calorimeter, as illustrated by Figs. 1 and 3, the frequency at which the minimum reflection factor of the calorimeter occurs can be controlled within limits by altering the input water temperature. This occurs because of the sensitivity of the dielectric constant of the water to temperature.[10] Such adjustment is not inconvenient when the water flow rate is high enough so that the thermal capacity of the calorimeter is sufficiently low.

(4) The water chamber of the calorimeter should be designed so that no air is entrapped and thorough water mixing occurs. Otherwise temperature fluctuations, hot spots and arcing may occur in the calorimeter.

Acknowledgment of appreciation is here given to the reviewers and W. Altar, of the Westinghouse Company for helpful suggestions.

[19] NDRC, MIT–RL Report T-9 and 189 (Library of Congress, Reference Department, Science Division, Navy Research Sec., Washington, D. C., 1943).

[20] Collins, ed., *Microwave Magnetrons* (McGraw-Hill Book Company, Inc., New York, 1948), Sec. 18.3; reference 1, Sec. 5.21; reference 18, Sec. 3.6.

[21] J. Booth, J. Chem. Phys. 19, 391–394 (1951).

Analysis and Synthesis of Optical Images

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The formation of an optical image by a lens is often treated so that the image appears as the double Fourier transform of the object, limited by the aperture of the lens and other stops. This treatment is here presented including details that are usually omitted in articles where an application of the treatment is the primary purpose.

Separate developments are presented that are most adaptable, respectively, to the configuration usually used with a microscope objective with a short object distance and a large image distance, and to the configuration normally used with a telescope objective.

THE formation of an image by a microscope is usually considered in two steps. The diffraction pattern formed by the object in the back focal plane of the objective lens is first calculated. The image is then the diffraction pattern formed in the image plane by the pattern in the focal plane. More than fifty years ago this concept was in use,[1] and subsequent writers have used it and modified it to their needs. Practically all of the writings on phase-contrast microscopy proceed in this manner, and a generalization of the method has been started in a recent series.[2]

The details of the development are invariably slighted, justifiably, to get on to applications at hand.

Some of the usually missing details are included in the following which deals with the simplest case, the one-dimensional object and image. The more practical case of two-dimensional images and objects is conceptually as simple, but more involved in detail. It can teach few principles that the one-dimensional case cannot.

HUYGENS' PRINCIPLE

Diffraction problems can be treated by Huygens' principle, in one of its more sophisticated forms, with approximations that may be practically unimportant.[3] For the three-dimensional, monochromatic case, each elementary area on a wave front acts as a point source putting out spherical waves that lead the parent wave in phase by $\frac{1}{2}\pi$, and the amplitude contributed by this element at some distant point at the proper later time is the wave amplitude of the parent wave multiplied by $(1+\cos\theta)dS/2\lambda r$. The factor $\frac{1}{2}(1+\cos\theta)$ is the "obliquity factor," θ being the angle the radius vector to the point from the element of area dS makes with the normal to dS in the direction of propagation of the parent wave; r is the length of this radius vector, and λ is the wavelength of the wave in question.

The foregoing statement is devised from a theorem that the wave amplitude at a point p is determined by[4]

$$w(p)=\frac{1}{4\pi}\int\int_S\left[w\frac{\partial}{\partial n}\left(\frac{e^{ikr}}{r}\right)-\frac{e^{ikr}}{r}\frac{\partial w}{\partial n}\right]dS, \quad (1)$$

where w is the complex amplitude of wave motion, with time dependence e^{-ikct} on a closed surface S surrounding p and containing no sources; r is the distance from S to p, and $\partial/\partial n$ means the component of the gradient in the direction of the inward normal of S; k is the propagation constant $2\pi/\lambda$. The wave velocity is c. The adaptation of this to Huygens' principle as stated follows:

Choose S so that it coincides with a surface that is perpendicular to the direction of propagation everywhere except in regions where the wave amplitude is practically zero, being cut off from sources by stops, or reduced to negligible value by great distance. Then Eq. (1) becomes

[1] A. B. Porter, Phil Mag. 11, 154 (1906).
[2] Elias, Grey, and Robinson, J. Opt. Soc. Am. 42, 127 (1952).
[3] J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill Book Company, Inc., New York, 1941), pp. 460–470 and references there given.
[4] B. B. Baker and E. T. Copson, *The Mathematical Theory of Huygens' Principle* (Oxford University Press, London, 1950), p. 24.

just the integral over this surface or wave front, the integrand being practically zero over the rest of the closed surface, and $\partial/\partial n$ is taken in the direction of propagation. If θ is the angle between this direction of propagation and the radius vector from S to p, then

$$w(p) = -\frac{1}{4\pi}\int\int_S ikwe^{ikr}\left[\left(\frac{1}{r}-\frac{1}{ikr^2}\right)\cos\theta+\frac{1}{r}\right]dS.$$

If p is many wavelengths from the wave front then $kr \gg 1$, and approximately

$$w(p) = (-i/2\lambda)\int\int_S (we^{ikr}/r)(1+\cos\theta)dS. \quad (2)$$

The exponential term represents the travel time of the wave from dS to p; $1/r$ is the reduction in amplitude due to spherical spreading, $-i$ is expressed in the rules as "advancing the Huygens" wavelets by $\frac{1}{2}\pi$, and the rest of the interpretation is evident.

The many approximations and assumptions that are required in proceeding from Eq. (1) to Eq. (2) restrict the use of Eq. (2) and the word statement of Eq. (2) above. These restrictions have been carefully dealt with[5] and will not be dwelt upon here.

The two-dimensional or cylindrical analogy of the foregoing is not so well known, although it is applied, in part, in many discussions of optical instruments. The statement of Huygens' principle is that each elementary length (if a cylindrical wave in three dimensions is contemplated, this length becomes an area) on a wave front acts as a source of wavelets that lead the parent wave by a phase difference $\frac{1}{4}\pi$, and the amplitude produced at some distant point after time r/c is the wave amplitude of the parent wave multiplied by $(1+\cos\theta)/2(\lambda r)^{\frac{1}{2}}$. The wave velocity is c, the obliquity factor is the same as with spherical wavelets, and the distance factor and the effect of the wavelength appear only to the one-half power.

The cylindrical problem is, in the general case, fraught with still more limitations than the three-dimensional problem,[6] but the monochromatic case is limited only to the same extent.

[5] Reference 4, pp. 45–67.
[6] Reference 4, p. 51.

The expression corresponding to Eq. (1) is

$$w(p) = \frac{i}{4}\int_l\left[w\frac{\partial}{\partial n}H_0^{(1)}(kr) - H_0^{(1)}(kr)\frac{\partial w}{\partial n}\right]dl; \quad (3)$$

where $H_0^{(1)}(kr)$ is the zeroth-order Hankel function of the first kind; l is a closed line surrounding p but not surrounding wave sources; w is the complex wave amplitude on l, and r is the distance from a point on l to p; $\partial/\partial n$ means differentiation in the direction of the inward normal of l.

Again choose the line or wave front perpendicular to the direction of propagation at each point as the part of l in Eq. (3) that contributes to the integral. With θ having the same meaning as before, Eq. (3) becomes

$$w(p) = (-i/4)\int_l [(w\cos\theta)(\partial H_0^{(1)}(kr)/\partial r) + ikwH_0^{(1)}(kr)]dl, \quad (4)$$

where the integration is to be carried out along the portion of l where the wave amplitude is nonzero.

Again r will be kept many wavelengths long so that $kr \gg 1$, and the asymptotic expression for $H_0^{(1)}(kr)$ may be used.[7] With no more approximation than was used in obtaining Eq. (2),

$$H_0^{(1)}(kr) \simeq (2/\pi kr)^{\frac{1}{2}}e^{i(kr-\pi/4)}. \quad (5)$$

Equation (5) substituted into Eq. (4) yields, after again invoking the approximation $kr \gg 1$,

$$w(p) = (-i/2\lambda^{\frac{1}{2}})\int_l w(1+\cos\theta)(e^{ikr}/r^{\frac{1}{2}})dl. \quad (6)$$

All is interpreted as with the spherical case, except that the term $e^{-i\pi/4} = (-i)^{\frac{1}{2}}$ may be said to amount to an advance of one-quarter wavelength in the effect at point p. This advance in phase of the Huygens' wavelets over the parent wave remained a mystery for many years. It results physically from the fact that the spherical and cylindrical solutions of the wave equation near the source do not behave like solutions for plane waves near a plane source. The wave motion is affected by the spreading of the waves.

The similarity between Eqs. (2) and (6) and their respective word statements is brought into focus if one notes that one always multiplies w

[7] Reference 3, p. 319.

by an obliquity factor, $\frac{1}{2}(1+\cos\theta)$, a time-lag factor e^{ikr}, and a combined factor which includes distance and the wavelength. This factor in the spherical case is $-i/\lambda r$, and in the cylindrical case $(-i/\lambda r)^{\frac{1}{2}}$. To get the amplitude contributed by the wavelet at p one also multiplies by the element of area, of the wave front dS in the spherical case, and by the element of length of the wave front dl in the cylindrical case.

IMAGE FORMATION BY A MICROSCOPE LENS

For simplicity consider only light that remains close to the optical axis of a cylindrical lens (Fig. 1), so that angles made with the axis can replace their sines and so that the cosines of these angles are either the first or the first two terms of the cosine series. Consider the object along x to be either illuminated or self-luminous so that along the axis is propagated a wave front that has amplitude $F(x)$ just above the object. Amplitude $F(x)$ may be a complex number if the phase is not everywhere the same, or if the phase is random, as it will be if the object is self-luminous.

Amplitude $F(x)$ is what can be learned about the object by looking down the optic axis. To find the resulting pattern in plane y, the back focal plane (Fig. 1), consider the effect at a point $y=\beta f$, where f is the focal length of the lens, due to $F(x)$ in region dx. The path from dx to y is away from x at angle β and thence through the lens to y. The effective distance to put into the amplitude factor is f since the wavelet does not converge on y until after passing the lens. The wave amplitude at y due to $F(x)$ in dx is

$$(-i/\lambda f)^{\frac{1}{2}}F(x) \times\exp\{ik[(d_o+f)(1+\tfrac{1}{2}\beta^2)-\beta^2 d_o-\beta x]\}dx.$$

The distance in the exponent is just the distance from x to y over the specified path to the approximation that $(\cos\beta)^{-1}=(1+\frac{1}{2}\beta^2)$, $\sin\beta=\beta$, and the extra "optical thickness" of the lens over its geometrical thickness is neglected. In the amplitude factor $(1+\cos\theta)$ has been replaced by 2.

Take $F(x)$ zero except near the optic axis. This is the usual situation when an object is viewed through a microscope. (The case with the object distance large as with the normal use of the telescope is best treated by another means as shown below.) Then the total wave amplitude in y is

$$G(y)=(-i/\lambda f)^{\frac{1}{2}}\int_{-\infty}^{\infty}F(x) \times\exp\{ik[d_o+f-\tfrac{1}{2}d_o\beta^2+\tfrac{1}{2}f\beta^2-\beta x]\}dx. \quad (7)$$

The limits of integration can be extended to $\pm\infty$, since $F(x)$ contributes nothing except near the axis. There is a small error introduced for values of β near the angle subtended by the lens $\pm\beta_1$. The image forming possibilities of the lens appear when the wave amplitude in a third plane z (Fig. 1) is calculated by applying Huygens' principle to the wave $G(y)$.

The contribution to the amplitude at z due to

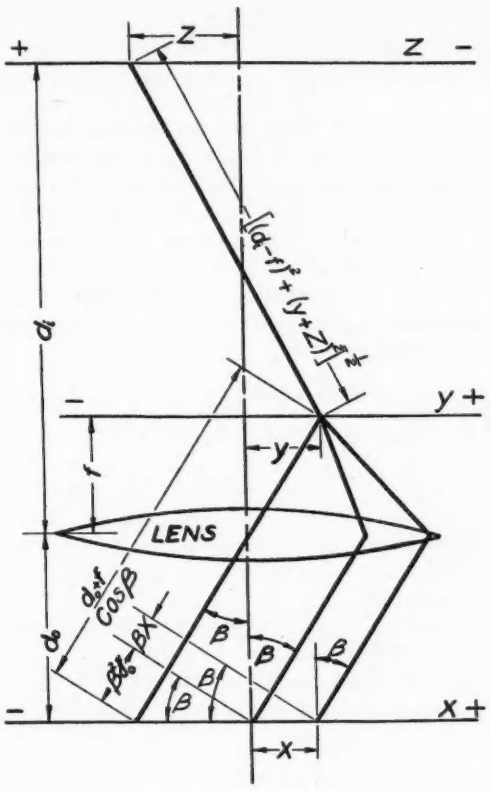

FIG. 1. Diagram of geometry considered in calculating diffraction patterns in the focal plane and the image plane of a cylindrical lens.

an element dy at y is

$$[-i/\lambda(d_i-f)]^{\frac{1}{2}}G(y) \times\exp\{ik[(d_i-f)^2+(y+z)^2]^{\frac{1}{2}}\}dy,$$

where d_i is the distance from the lens to the z plane. The sign of z is chosen as shown in Fig. 1. For the effect of distance (d_i-f) alone is used in the amplitude factor, but the exact distance from point y to point z is carried in the more sensitive phase term.

To the approximation applied above to the distances in the phase terms, the total wave amplitude in z is then

$$J(z)=\left(\frac{-i}{\lambda(d_i-f)}\right)^{\frac{1}{2}}\int_{-y_1}^{y_1}G(y) \times\exp\left\{ik\left[d_i-f+\frac{z^2+y^2}{2(d_i-f)}+\frac{zy}{d_i-f}\right]\right\}dy. \quad (8)$$

The limits $\pm y_1=\pm\beta_1 f$ are the limits set by the lens aperture.

If Eq. (7) is substituted in Eq. (8) and terms not dependent on a variable of integration are arranged outside their respective integral signs, and if y is replaced by βf, Eq. (8) becomes

$$J(z)=\frac{-i}{\lambda}\left(\frac{f}{d_i-f}\right)^{\frac{1}{2}}\exp\left\{ik\left[d_o+d_i+\frac{z^2}{2(d_i-f)}\right]\right\} \times\int_{-\beta_1}^{\beta_1}\left\{\int_{-\infty}^{\infty}F(x)e^{-ik\beta x}dx\right\} \times\exp\left\{ik\left[\frac{\beta^2}{2}\left(f-d_o+\frac{f^2}{d_i-f}\right)+\frac{\beta fz}{d_i-f}\right]\right\}d\beta. \quad (9)$$

Equation (9) holds some resemblance to a double Fourier transform, and the resemblance will be improved if the coefficient of β^2 in the exponential of the integrand is zero. Given f and d_o, choose d_i to bring this about. This requires $f-d_o+f^2/(d_i-f)=0$, which may be rearranged to

$$1/f=1/d_o+1/d_i, \quad (10)$$

the lens equation.

Except for the noninfinite limits on one of the integrals and a constant factor, the resemblance to a double Fourier transform will be complete if $k\beta$ is chosen as the second variable of integration and if $(d_i-f)/f$ is set equal to M. Quantity M turns out to be the usual magnification since, subject to Eq. (10), $M=d_i/d_o$. These substitutions change Eq. (9) to

$$J(z)=-iM^{-\frac{1}{2}}\exp\{ik[d_o+d_i+z^2/2(d_i-f)]\} \times\left[(1/2\pi)\int_{-k\beta_1}^{k\beta_1}\left\{\int_{-\infty}^{\infty}F(x)e^{-ik\beta x}dx\right\} \times e^{ik\beta z/M}d(k\beta)\right]. \quad (11)$$

The factor in brackets is the double Fourier integral. If only the limits to the second integral were $\pm\infty$, the bracket would have the value $F(z/M)$, i.e., the original pattern reproduced in z but magnified by M. The inversion of the image was taken care of by the choice of signs in Fig. 1. The amplitude factor indicates that the amplitude is reduced by the factor $M^{-\frac{1}{2}}$; therefore the intensity is reduced by $1/M$, which is correct for the image produced by a cylindrical lens. The exponential outside the bracket represents the phase lag introduced by the passage from x to z. It just corresponds to the lag incurred in distance (d_i+d_o) except for the second-order term that represents the added phase lag due to the greater distance for points off the axis. This is just the extra phase lag that one would calculate by geometric optics. The optical distance from a point x to its conjugate point in z is to the above approximations $(d_o+d_i)(1+z^2/2d_i^2)$, which can be expressed, with the help of the lens equation (10), as $d_o+d_i+z^2/2(d_i-f)$.

The lens equation expresses just the condition required to render a factor of Eq. (9) a double Fourier transform and, hence, make the wave amplitude in z the "image" of that in x. It is comforting that the resulting image has the proper magnification and intensity. It is unlikely that this derivation of the lens equation will supersede the more usual one.

The cutoff caused by the aperture cuts out Fourier components of $F(x)$ of wave number $k\beta$ greater than $k\beta_1$, and hence details of $F(x)$ carried out by these components will not appear in the image. The Rayleigh criterion of resolution for the cylindrical lens results from application of this high wave-number cutoff by the lens aperture when $F(x)$ consists of two closely

spaced line sources located at $x=a$ and $x=b$. Then

$$F(x)=\delta(a)\phi_a+\delta(b)\varphi_b,$$

where $\delta(x)$ is a delta-function and φ_a and φ_b are terms of unit magnitude to set the phases of the signals. The separation $(b-a)$ that matches the Rayleigh criterion will be different for random phases, which one gets from self-luminous sources, and coherent phases, which result when the two lines are illuminated by the same source. In the latter case, the relative phase of light scattered by the two objects will affect resolution.

Various spectacular experiments have been described that involve placing stops in the plane y to perform various "filtering" operations.[1]

One is tempted to claim that in the focal plane of the objective is formed essentially the Fourier transform of the object function $F(x)$.

This cannot be the case; for if it were, then a second lens would produce the inverse transform in its focal plane, and the result would be that two lens in tandem would produce an in-focus image in the focal plane of the second lens regardless of the other distances, the lens separation and the distance to the object. This is known to be not the case experimentally and on the basis of geometrical optics.

The fallacy is that it is not the Fourier transform of the object that is produced in the focal plane, but $G(y)$ of Eq. (7), which has an amplitude proportional to that of the Fourier transform, but differs in phase by $ky^2(f-d_o)/2f^2$ in addition to a constant phase difference $k(d_o+f)$.

OPTICAL HARMONIC ANALYZERS

One is also tempted to design a device for optically analyzing signals recorded as sound-on-film. For a variable density sound track one needs only to illuminate the part of the track to be analyzed with monochromatic light, form a reduced image of the track with a low power microscope objective and then "analyze" the image with a cylindrical lens. In the focal plane of the lens will appear the spectrum of the reduced image; this is, $G(y)$, which gives the amplitide of the Fourier transform of $F(x)$. This may be sufficient for certain practical needs. However, it is the amplitude of $G(y)$, not the intensity, that is usually wanted. This can be remedied by suitably calibrating the device that responds to the intensity of $G(y)$—photocell, photographic plate, etc. For a usual sound-on-film record, however, $F(x)$ would not be a replica of the original signal. It is the intensity, not the light amplitude through a variable density sound-on-film track, that varies with the electrical signal. Consequently, to get a visual analysis of the signal the sound-on-film would have to be recorded through a network whose output is the square root of the magnitude of the input signal and arranged to retain the sign of the input signal.

The purpose of the image of the film reduced in size by a microscope objective is to render the wave numbers of the pattern large enough to make the analysis easier. The reduction should be in a ratio that will render in the reduced image the wave number corresponding to the highest frequency in the signal to be analyzed about $1/3\lambda$.

A cylindrical lens will smear out a variable area sound-on-film record to a reasonable replica of variable density sound-on-film. Hence, the analysis is possible, but requires more than just a lens.

TELESCOPE IMAGES

The configuration of the above optical systems corresponds to small objects placed just beyond the focal plane of the lens, the configuration usually encountered with microscopes. For large far-away objects, the configuration for the normal use of a telescope, the integral in Eq. (7) does not represent the light at the focal plane of the lens. The construction of $G(y)$ depended on light leaving all points of the object at angle β entering the lens. This is not the case in the normal use of a telescope objective. It is possible to modify the limits on the integrals of Eqs. (7) and (9) to represent the case of the telescope, but this becomes so artificial that little or nothing is gained thereby.

For the telescope configuration it is better to go through the inverse process. First calculate the diffraction pattern that the object produces in the front focal plane of the objective, and then the pattern produced in the image plane by this pattern.

In this development let x' represent position in the object plane, y', position in the front focal plane of the objective lens, and z' the plane behind the lens that will be identified with the image plane. Use the same symbols, d_o for object distance, d_i for image distance, and f for focal length. In Fig. 1 the plane z is here taken over by x', y by y', x by z'.

The object in the x' plane puts out a wave front $F'(x')$. The wave pattern in the y' plane due to $F'(x')$ is calculated just as the pattern in the z plane was calculated from $G(y)$ in the case of the microscope. The sign of y' and z' is opposite that of x'.

This pattern $G'(y')$ is

$$G'(y')=\left[\frac{-i}{\lambda(d_o-f)}\right]^{\frac{1}{2}}\int_{-\infty}^{\infty}F'(x')$$

$$\times\exp\left\{ik\left[d_o-f+\frac{1}{2}\frac{x'^2+y'^2}{d_o-f}+\frac{x'y'}{d_o-f}\right]\right\}dx'. \quad (12)$$

The limits on the integral are overstatements unless the object is confined to a small range of azimuth. Then $G'(y')$ produces in a plane located distance d_i behind the lens, the pattern

$$J'(z')=\left(\frac{-i}{\lambda f}\right)^{\frac{1}{2}}\int_{y_2'}^{y_3'}G'(y')$$

$$\times\exp\left\{ik\left[(d_i+f)\left(1+\frac{y'^2}{2f^2}\right)-\frac{y'^2d_i}{f^2}-\frac{z'y'}{f}\right]\right\}dy',$$

or with $\beta'=y'/f$ and $M'=f/(d_o-f)$,

$$J'(z')=(-iM'^{\frac{1}{2}}/\lambda)e^{ik(d_o+d_i)}\int_{+\beta_3'}^{\beta_3'}\left\{\int_{-\infty}^{\infty}[F'(x')\right.$$

$$\left.\times\exp\{ikx'^2/2(d_o-f)\}]e^{ik\beta'z'M'}dx'\right\}$$

$$\times\exp ik\{\tfrac{1}{2}\beta^2[f-d_i+f^2/(d_o-f)]-\beta'z'\}d\beta'. \quad (13)$$

If the lens equation is satisfied the exponents in the integrand containing β'^2 vanish and

$$J'(z')=-iM'^{-\frac{1}{2}}e^{ik(d_o+d_i)}$$

$$\times\left[(1/2\pi)\int_{k\beta_2'}^{k\beta_3'}\left\{\int_{-\infty}^{\infty}[F'(x')\exp\{ikx'^2/2(d_o-f)\}]\right.\right.$$

$$\left.\left.\times e^{+ik\beta'z'M'}d(x'M')\right\}e^{-ik\beta'z'}d(k\beta')\right]. \quad (14)$$

Except for the constant phase factor $ie^{ik(d_o+d_i)}$ and the amplitude factor, $M'^{-\frac{1}{2}}$, $J'(z')$ is nearly a double Fourier transform of not $F'(x')$ but of $F'(x)\exp(ikx'^2/2(d_o-f)$, which has the same amplitude as $F'(x')$ but differs in phase. This represents only the extra distance for object points off the axis, but is of no consequence in the usual use of a telescope where there is no coherent phase relation between separated points on the object.

The amplitude of the image is properly multiplied by $M'^{-\frac{1}{2}}$ and linear dimensions of the image by M'. A displacement in the object of $\Delta x'$ corresponds to $\Delta z'=M'\Delta x'$.

Reproduction of the image is limited by the limits β_2 and β_3 imposed by the lens aperture. These limits will be different for different parts of the image. Only their difference is approximately constant, $\beta_3-\beta_2=A/F$, where A is the width of the lens. The usual minimum angle of resolution can be derived from this cutoff if two closely spaced sources are considered as the object.

It is noteworthy that stops designed to "filter" the Fourier components of the object function $F'(x')$ before reproduction as $J'(z')$ should be placed in the front focal plane of the lens. A pair of slits, for example, at $y'=\pm y_4'$ let through just the wave number component of the object near $k\beta_4=ky_4/f$. A line (or point) source in the object is just an impulse or a delta-function which contains components of all wave numbers evenly distributed. The slits mentioned pass only the one component, and this reconstructed function is one sinusoidally varying in z' with a linear "period" (in z') of $2\lambda f/d=\lambda f/y_4$, where d is the slit separation. This is the usual interference pattern of two slits. The intensity goes through zero and maxima with half this period, or $\lambda f/d$.

When such interference patterns are considered for measuring stellar separations (two stars being two incoherent delta-functions closely spaced in x') or stellar diameters, the slits are usually considered just in front of the lens. There is little practical difference when the wave front from the object is practically plane, and the focal length is many wavelengths long. The only difference is that with the slits in the front focal plane, the interfering waves in the image plane

are plane waves, while with the slits at the lens, the interfering waves are cylindrical.

The same differences hold for sources not at infinity; but the period of the fringes is longer with the slits at the lens, and the expression stated for this period holds only if the slits are at the front focus.

If one tries to apply an analog of one of the preceding schemes to a configuration that produces a virtual image, he will meet with frustration. This emphasizes the fact that there is no light at a virtual image, and this application of Huygens' principle does trace the wave motion, the light itself, through the optical system.

A system in which a virtual image is considered usually has other lenses in it. Similar applications of Huygens' principle can trace the light through the whole system to some final plane where a real image is formed.

The Prenatal History of Electrical Science

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The history of electricity was at first the history of the "amber effect," that is, of the "attraction" for light objects displayed by rubbed amber. This effect was well known in antiquity and was coupled and confused with the attraction for iron by the loadstone. The earliest explanations of these phenomena were anthropomorphic, being followed, in classical antiquity, by teleological organismic explanations and also materialistic mechanistic explanations. In the 15th century the lack of a directive (north-south) property in amber was noticed. By the 16th century several substances exhibiting the amber effect were known. From this time onward, attempts were made to provide a theory for the amber effect independently of magnetic phenomena. Effluvium theories, so important in the later history of electricity, first began to develop in the 16th century. The present paper covers the period up to the work of Gilbert in 1600.

ELECTRICITY as a science may seem to have little history of significance prior to the appearance of William Gilbert's *De Magnete* in 1600. However, in this prenatal period, in which electrical knowledge was so meager, it is a comparatively simple matter to trace the main recorded notions and speculations, and, for this very reason, to bring out rather clearly and sharply some important main characteristics of early modes of approach to an understanding of natural phenomena. The relative simplicity of the prenatal stage of electrical science, and its many connections with early philosophy and literature, make its study useful and attractive to the nonscientist. And to scientists and nonscientists alike, familiarity with the outlook at the beginnings of a science may be of importance in evaluating current thought in various fields of knowledge and activity where similar outlooks are still predominant today.

The study of ancient and medieval science is also of value in connection with the cultural importance of the Scientific Revolution of the 16th and 17th centuries. Herbert Butterfield, who is a general historian and not primarily a historian of science, has said of that Revolution:

> . . . it outshines everything since the rise of Christianity and reduces the Renaissance and Reformation to the rank of mere episodes, mere internal displacements, within the system of medieval Christendom.[1]

The Literary Renaissance and the Protestant Reformation were, in an important respect, attempts to restore *old* outlooks and things that

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[1] H. Butterfield, *The Origins of Modern Science, 1300-1800* (Macmillan Company, New York, 1951), p. viii.

had been forgotten, whereas the Scientific Revolution, as A. N. Whitehead[2] has said, represented "the most intimate change in outlook which the human race had yet encountered." But to be able to appreciate and understand any *revolutionary change* one obviously must have some knowledge of the situation before the revolution as well as afterward. A critical point plotted on a graph cannot be recognized as such—much less interpreted—unless the curve on both sides of the point is also shown.

The earliest known electrical phenomena.—It is commonly, and correctly, pointed out in histories of science that the people of antiquity were familiar with four types of phenomena that today we recognize as electrical in character. These are: (1) the lightning discharge, which was generally thought by the ancients to be a sort of fire or "burning vapor," perhaps sulfurous in character; (2) the discharge with which the so-called torpedo fish stuns its prey, known to the early Egyptians and later recorded by Greek and Roman naturalists; (3) "St. Elmo's fire," a pale glow sometimes seen on the tips of pointed objects during stormy weather, described by the ancient Romans in their military camps as "spears with flames at their points"; (4) the property of attracting small objects shown by rubbed amber, often referred to as the *amber effect*.[3]

But that there was any connection between these four kinds of phenomena was not recognized in antiquity or, indeed, until comparatively modern times. It was solely the amber effect that provided the early basis for the great body of knowledge constituting modern electrical science and technology.

AMBER AND THE AMBER EFFECT IN ANTIQUITY

The substance that we call amber was used by the ancients as a jewel and for ornamentation; trade in it probably existed even in prehistoric times. Amber beads have been recovered from ancient tombs. There are frequent references to amber in Greek literature, as early as in the *Odyssey* of Homer (before the 9th century B. C.), and later, for instance, in the plays of Aeschylus, Sophocles, and Euripides (5th century B. C.). Among the Greeks amber came to be called *elektron* (ἤλεκτρον) which was the name they also gave to native silver-gold alloy, because the color of these substances was remindful of the pale yellow of sunlight.

Early Speculations about the Origin and Sources of Amber

The early knowledge of the amber effect was colored by current speculations concerning the origin and sources of amber. There had been many mythical tales: amber was the congealed "urine of the wild beast called the lynx, male or female"; it was the tears of the poplar trees into which the Heliades had been turned as they lamented the death of their brother, Phaëton; it was an unctuous sweat produced on the soil by the fierce rays of the sun and carried off by the tides of the ocean. Pliny (A.D. 1st century), in ending his account of "Amber: the many falsehoods that have been told about it," wrote:

> But the one that has surpassed them all is Sophocles, the tragic poet [5th century B. C.]; a thing that indeed surprises me, when I only consider the surpassing gravity of his lofty style, the high repute that he enjoyed in life, ... According to him, amber is produced in the countries beyond India, from the tears that are shed for Meleager, by the birds called "meleagrides." Who can be otherwise than surprised that he should have believed such a thing as this, or have hoped to persuade others to believe it? What child, too, could possibly be found in such a state of ignorance as to believe that birds weep once a year, that their tears are so prolific as this, or that they go all the way from Greece, where Meleager died, to India to weep?[4]

One aspect of the development of Greek science from about 600 B. C. onward was the shift toward nonmythical descriptions and interpretations. Thus Aristotle (4th century B. C.), apparently knowing that amber is frequently found near bodies of water, gave a "physical" explanation of why it occurs in solid form: "The heat is

[2] A. N. Whitehead, *Science and the Modern World* (Macmillan Company, New York, 1925), p. 3; also Pelican Mentor Book M28, p. 2.

[3] Interesting accounts of ancient knowledge and speculations concerning these various phenomena are given by Gaius Plinius Secundus (Pliny "the Elder"), *Historia Naturalis* (A. D. 1st century), translated under the title *The Natural History of Pliny* by J. Bostock and H. T. Riley (Henry G. Bohn, London, 1855–57). See entries, "Lightning," "Torpedo," and "Amber" in the Index, Vol. VI.

[4] Pliny, reference 3, Bk. XXXVII, chap. 11.

driven out of it by the cold of the river and causes the moisture in the amber to evaporate with it."[5] As evidence that the amber occurred originally in liquid form and afterward solidified, he cited "the animals enclosed in it."[5] Of an ant enclosed in amber, the Roman epigrammatist, Martial (A. D. 1st century) was later to write:

> So she that was in life despised,
> In death preserved, is highly prized.

Apparently amber containing insects was considered desirable jewelry.

The view that amber somehow came from trees was quite common in antiquity. According to Pliny,

> Our forefathers, too, were of the opinion that it is the juice of a tree, and for this reason gave it the name of "succinum": and one great proof that it is the produce of a tree of the pine genus, is the fact that it emits a pine-like smell when rubbed, and that it burns, when ignited, with the odor and appearance of torch-pine wood.[3]

Although doubt about the origin of amber was to remain for more than 17 centuries to come, Aristotle and Pliny together had provided an explanation little different from the modern one. For, in the 19th century, it was shown that amber is the fossil resin of pine trees which probably have been extinct for many millions of years. Moreover, it is frequently found on or near seacoasts, often entangled in seaweed washed up after storms. The Latin name for amber mentioned by Pliny is the basis for the modern terms, "succinic acid" and "succinite."

The Amber Effect

It may have been known in remote antiquity that a piece of amber which has been rubbed—perhaps first accidentally against the skins with which early man clothed himself—"attracts" to itself the hairs of fur and other light objects. Yet of this amber effect there is no extant written record until the early part of the 4th century B. C., when Plato, in one of his dialogues, the *Timaeus*, wrote:

> Moreover, as to the flowing of water, . . . and the marvels that are observed about the attraction of amber and the Heraclean stones—in none of these cases is there any attraction; but he who investigates rightly, will find that such wonderful phenomena are attributable to the combination of certain conditions—the nonexistence of a vacuum, the fact that objects push one another round, and that they change places, passing severally into their proper positions as they are divided and combined.
>
> Such as we have seen, is the nature and such are the causes of respiration—the subject in which this discussion originated. . . .[6]

In this first known reference to the amber effect, it is to be noticed that (1) the reference to the effect is casual, indicating that Plato regarded its "marvels" as already well known to his readers; (2) the mention of it is only incidental, in connection with an inquiry into the "causes" of human respiration; (3) a physical explanation of the effect is offered but merely in an attempt to demonstrate that it is only one of a number of phenomena explainable on the same principles as is respiration; (4) the implication is that the effect is closely related to that of the Heraclean stone, or natural magnet. As we shall see, the important distinctions between the amber and loadstone phenomena did not become a subject for serious inquiry until some 20 centuries later.

Many modern writers credit Thales of Miletus (6th century B. C.) with having observed and discussed both the amber effect and the natural magnet. Such credit, however, is based entirely on hearsay, for none of Thales' writings is extant, and it is not even certain that he left any writings. The assertion that he knew of both effects appears in a book written some eight centuries after his time: "But Aristotle and Hippias say that he [Thales] attributed souls also to lifeless things, forming his conjecture from the nature of the magnet, and of amber."[7] What Hippias may have said cannot be verified, for his writings are also lost. As for Aristotle, he

[5] *Meteorologica* (4th century B. C.), pp. 388–89. All of our references to Aristotle are to the eleven volume English translation of his works edited by W. D. Ross (Oxford University Press, London, 1908–31). For a useful index of specific topics and terms treated by Aristotle, see T. W. Orgon, *An Index to Aristotle* (Princeton University Press, Princeton, 1949).

[6] *The Dialogues of Plato*, translated by B. Jowett (Random House, Inc., New York, 1937), third ed., Vol. 2, p. 57.

[7] Diogenes Laërtius, *The Lives and Opinions of Eminent Philosophers* (A. D. *c* 3rd century), translated by C. D. Yonge (G. Bell and Sons, London, 1901), p. 15.

wrote in his *De Anima* (p. 405): "Thales, too, to judge from what is recorded about him, seems to have held soul to be a motive force, since he said that the magnet has a soul in it because it moves iron." Aristotle made three references to the origin of amber, in his *Meteorologica* (pp. 388–89), but nowhere did he mention the amber effect, much less Thales' possible knowledge of it.

There is one reference to the amber effect that concerns a practice which may have originated long before the time of Plato or even of Thales. According to Pliny, it was reported by Nicias, concerning whom nothing is known except that he wrote about "stones,"

> . . . that in Syria the women make the whirls [wharves] of their spindles of this substance [amber], and give it the name of "harpax" ["dragger," or "clutcher"], from the circumstance that it attracts leaves toward it, chaff, and the light fringes of tissues.[3]

That the amber wharve, being continually rubbed during the spinning, picked up and held to it nearby objects, is something that could hardly have escaped the conscious attention of the spinner. Thus the existence of the amber effect may have been known for centuries preceding Plato. It may also have been mentioned or described repeatedly, for it is well known that only a minute part of Greek and other ancient writings has survived. Nevertheless, it is in the *Timaeus* (4th century B. C.) that the first known reference to the effect appears.

The Earliest References to the Magnet

Although our concern here is mainly with the amber effect, it and magnetism were so frequently mentioned together and confused by early writers that one cannot completely separate their histories.

As Aristotle indicated in *De Anima*, Thales (6th century B. C.) may have known about the natural magnet, later to be called the *loadstone*, and the fact that it "moves iron." However, the first known reference to the phenomenon appears in a fragment from a lost play of Euripides called the *Oeneus* (5th century B. C.):

> . . . like the Magnesian stone, he draws opinion to himself, and then changes it again.[8]

[8] This fragment is preserved in the dictionaries of two Byzantine lexicographers, Photius (A. D. 9th century) and Suidas (A. D. 10th century), and is quoted in *Euripides Opera Omnia* (Glasgow, 1821), Vol. VII, p. 640, fragment v of the *Oeneus*.

About a century later, Plato wrote, in his dialogue called the *Ion:*

> The gift which you possess of speaking excellently about Homer is not an art, but, as I was just saying, an inspiration; there is a divinity moving you, like that contained in the stone which Euripides calls a magnet, but which is commonly known as the stone of Heraclea. This stone not only attracts iron rings, but also imparts to them a similar power of attracting other rings; and sometimes you may see a number of pieces of iron and rings suspended from one another so as to form quite a long chain: and all of them derive their power of suspension from the original stone. In like manner the Muse first of all inspires men herself; and from these inspired persons a chain of other persons is suspended, who take the inspiration. For all good poets. . . .[9]

These first references to the magnetic effect, like those to the amber effect, mention the phenomenon only incidentally, and here for purposes of simile and metaphor. But in resorting to such figures of speech to clarify or emphasize a topic, a competent writer will not cite a phenomenon that is likely to be unfamiliar to many of his readers. Clearly then, these first references mark, not the times of discovery, but merely the earliest periods for which we can be sure that the phenomena were already well known to many people.

"Discovery" and "Effective Discovery"

Any discussion of the first records of a phenomenon obviously provides an opportunity to correct some current misconceptions about the nature of *discovery* in a science. For instance, one can point out that countless people in antiquity may have seen the movement of light objects toward rubbed amber, but without being conscious that anything unusual had occurred. However, this does not constitute *discovery* of the phenomenon in any useful sense of the term. At the least, a discovery involves conscious, intelligent observation such as was reported of the Syrian spinners. But even such conscious, intelligent observation may not constitute an *effective discovery*. A scientific discovery should be regarded as effective only if it has a distinct effect on future progress in the science or in,

[9] Reference 6, Vol. 1, p. 289.

say, some practical art. As this history proceeds, we shall see that it is not easy to decide just when the effective discovery of the amber effect occurred.

The notion of *effective discovery* obviously can be used to advantage in expanding on statements such as the following: ". . . it is invariably true, no matter what the period, that if an investigator publishes a fundamentally new fact, sooner or later voices will be heard claiming priority either for themselves or for a third party, because it is alleged that they 'really' had made the discovery earlier";[10] ". . . a principle in the strategy of science, namely, that the time must be ripe for a scientific discovery to be regarded as significant."[11] The notion may also be applied, and in these instances devastatingly, to assertions such as the following: "Not one single new idea has appeared in any book published during the 20th century"; "Within college walls it does not matter whether you live in Egypt or Athens or in the 18th or 20th century; the world of ideas is timeless." These two statements typify the point of view of those subscribing to what has aptly been called "sterile scholasticism."

"Lyncurium"

Theophrastus (4th century B. C.), in the oldest extant treatise on minerals, his *On Stones*, wrote of

> . . . a stone called lyncurium . . . [which has] an attractive power like that of amber, and is said to attract not only straws and small pieces of sticks, but even copper and iron, if they are beaten into thin pieces. This Diocles affirms. . . . Amber also is a stone: it is dug out of the earth in Liguria, and has, as the before mentioned, a power of attraction. But the greatest and most evident attractive quality is in that stone [the Heraclean stone] which attracts iron. But this is a scarce stone, and found in but few places; it ought, however, to be ranked with these stones, as it possesses a like quality.[12]

Here we see Theophrastus coupling amber and the natural magnet together, just as Plato did. But Theophrastus brings in something new by adding to the group another "stone" which he calls "lyncurium" and describes elsewhere as of very solid texture and used as a material on which to engrave seals. As to just what he meant by "lyncurium," one cannot be sure; but from his description, modern mineralogists think it may have been topaz, tourmaline, or zircon. There were specimens of many such gems and minerals in Aristotle's enormous natural history collection, to which Theophrastus had access, for he was the pupil and colleague of Aristotle and the latter's successor as head of the Lyceum. Yet he apparently did not try to see for himself whether any of these specimens exhibited an "attractive power"; he seemed content to report what others had said.

Theophrastus seems also to have been the first to record the suggestion that thin pieces of metal are among the light objects subject to attraction. But here again there is no mention of any attempt on his part to see whether the leaf-metal would be attracted by rubbed amber as well as by "lyncurium." Leaf-metal as a sensitive detector of electrification did not come into general use until the 18th century.

Concerning both "lyncurium" and the attraction for leaf-metal, Pliny later had this to say:

> The pertinacity that has been displayed by certain authors compels me to speak of lycurium next; for even those who maintain that it is not a variety of amber, still assure us that it is a precious stone. . . . The color of it, they inform us, like that of some kinds of amber, is a fiery hue, and it admits, they say, of being engraved. They assert, too, that this substance attracts to itself not only leaves or straws, but thin plates of copper or of iron—a story that even Theophrastus believes on the faith of a certain Diocles.
>
> For my own part I look upon the whole of these statements as untrue, and I do not believe that in our time there has ever been a precious stone with such a name as this. . . .[13]

How Pliny reached this conclusion, he does not say, but almost certainly it was not through any experimentation. Of Theophrastus, one could have expected experimental verification of his statements, for Theophrastus was a scientist whose writings on botany, for example, were never excelled until the Renaissance. Pliny, on the other hand, was a state official, a man of

[10] M. von Laue, *History of Physics*, translated by R. Oesper (Academic Press Inc., New York, 1950), p. 7.

[11] J. B. Conant, *Science and Common Sense* (Yale University Press, New Haven, 1951), pp. 171, 119.

[12] Theophrastus, περι λιθων, translated by John Hill under the title *History of Stones* (London, 1774), pp. 125–137.

[13] Pliny, reference 3, Bk. XXXVII, chap. 13.

affairs, and an omnivorous reader who is said to have collected material from some 2000 volumes for his *Historia Naturalis*,[3] a great encyclopedia of ancient knowledge on almost every known subject. The importance of the *Historia Naturalis* to us is that it was never lost to the West, even during the Dark Ages, and thus probably influenced many subsequent commentators and investigators of the amber effect.

Of the "attractive power" of amber itself, Pliny wrote:

> When a vivifying heat has been imparted to it by rubbing it between the fingers, amber will attract chaff, dried leaves, and thin bark, just in the same way that the magnet attracts iron.[14]

Note the assertion that the heat produced by rubbing is the factor of primary importance. The fact that rise in temperature and electrification by rubbing always occur together was to plague experimenters long after electricity reached the status of a science. Not until the 18th century with the discovery of ways to electrify a body other than by rubbing, did it begin to be clear that rise in temperature plays no essential role in triboelectrification.

Pliny cited a geographical writer by the name of Philemon as having *erroneously* asserted that "electrum [amber] does not yield a flame." If by "flame" was meant an electric spark, as has been suggested by some modern commentators, then it would seem that Philemon must have heard reports from others of sparks obtained from rubbed amber and that he regarded these reports as incorrect. Even if they were correct, the discovery should be regarded as completely ineffective, for there is no known, subsequent mention of electric sparks until late in the 17th century. The more plausible explanation is that Philemon thought that amber would not burn, whereas Pliny knew this to be untrue.

Early Explanations

It is noticeable that even the first records of the amber effect go beyond a mere description of the observations. What the Syrian spinners doubtless *saw* was a bit of thread, say, near the rubbed amber at one moment and apparently in contact with it a moment later. But to imply that the amber "clutches," "drags," or "attracts" the thread is not a description of what was observed but an *inference* from it. Thus, from a very early date, there was an attempt, consciously or unconsciously, to *explain* the observations rather than merely to describe them.

This urge to explain is of course likely to accompany any observation that is new or surprising. One tries to render the observation more intelligible by making it fit in with more familiar experiences, thus showing that what at first seems to be new and surprising is, after all, not new but is common experience perhaps present in the phenomenon in a disguised form. Since most of early man's knowledge was concerned intimately with his own personal experiences, his natural tendency was to interpret all phenomena in mythical, anthropomorphic terms. We have seen examples of such interpretations in the early speculations on the origin of amber. Of the amber effect itself, a possible explanation was, to use a metaphor from the ancient hymns of Orpheus, that objects come to the amber "as the bride to the embraces of her spouse." Again, one explanation given for the observed differences in strength of loadstones was that there are two kinds, male and female, those of the latter sex being the weaker or even destitute of attractive power.[3]

From about 600 B. C. onward the early "nature myths" were gradually replaced in the thinking of the Greek philosophers by the sorts of explanation termed *scientific*. In physics and cosmology, two types of such explanation developed. One was teleological in character and is sometimes referred to as "organismic," because the basic principle is the analogy between physical phenomena and the behavior of organisms.[15] For example, one early organismic hypothesis concerning amber was that rubbing developed in it a "longing" or "need" for light objects, as if they served as "food" for the amber; and to satisfy this "need" the amber caught its "prey" by projecting arms, as do many living things. Another such explanation, favored by Pliny,[16] was that all objects, inanimate as well as animate, possess "states, known to the Greeks by

[14] Pliny, reference 3, Bk. XXXVII, chap. 12.

[15] P. Frank, Am. J. Phys. 15, 214 (1947).

[16] Pliny, reference 3, Bk. XX, chap. 1.

the respective appellations *sympathia* and *antipathia*," and that "to these states . . . we are indebted for the first principles of all things"; thus objects that are mutually in "sympathia" would tend to unite or to interact, whereas those in "antipathia," or "natural contrariety," would exhibit the opposite tendency. Plato's explanation in the *Timaeus*[6] was also teleological, although less patently so.

The second type of physical explanation developed among the Greeks may be termed "materialistic" or "mechanistic." Such a hypothesis for the action of the loadstone was advanced in the century following Plato, by the Greek philosopher Epicurus (4th century B. C.). His physics, which was based upon the doctrine of Democritus (5th century B. C.) regarding "the atoms and the void," was afterward expounded with amazing effectiveness by the Roman poet Titus Lucretius Carus (1st century B. C.), in "that greatest of all philosophic poems," *De Rerum Natura*. The following passage from the poem pertains to the loadstone's attraction for iron:

> First, from this stone there must stream a shoal of seeds in a current
> Driving away with its blows all the air 'twixt itself and the iron.
> Soon as this space is made void and a vacuum fashioned between them,
> Instantly into that void the atoms of iron tumble headlong
> All in a lump; thus the ring itself moves bodily forward.
> Naught that exists has its ultimate parts in more intricate tangle,
> Naught is more closely compressed than the substance of obdurate iron
> So very dreadfully cold. Therefore less your amaze when I tell you
> That from constituent parts like these large numbers of atoms
> Cannot break forth from the iron and into the void be transported
> Save by the advance of the whole of the ring. This accordingly happens;
> Therefore the ring moves on till it touches the stone and thereunto
> Cleaves, attached by invisible bonds. . . .[17]

The remaining verses pertaining to the magnet —all are in the latter portion of Book VI—are not reproduced here. The student of the early history of magnetism will find much of interest in them, especially in verse 41, which is believed to be the first record of the phenomenon of magnetic *repulsion* and also of the observation that bits of iron in a brass bowl will move when a loadstone is moved around underneath the bowl.

Incidentally, this poem of Lucretius has an obvious appeal for students of the humanities who are in a physical science course, for it provides an unusually good illustration of how poetry, or any art for that matter, may function in the dissemination of scientific knowledge. Any art may function (1) *esthetically* in or for itself, and (2) *philosophically* as a means for conveying information about some other field of knowledge, for instance, some theory of a natural science, or of history, or of theology. In *De Rerum Natura* the philosophic function dominates. Lucretius found "modern" doctrines and ideas in the Epicurean philosophy that he thought would bring enlightenment and comfort to all people. But Epicurus not only had written in Greek, but had employed the technical language of the specialist. So Lucretius, in addition to using the Latin of Rome, recast the Epicurean views into the emotive language of poetry, thus immensely increasing their general appeal and meaningfulness. Today, *De Rerum Natura*, in its philosophic function, is, of course, to be regarded as strictly dated and solely of historical interest. But as sheer poetry, and to the extent that it increases one's awareness and appreciation of nature in all its richness and beauty, the poem is as good today as at the time it was written, some 20 centuries ago.[18]

[17] W. H. Brown, *Lucretius on the Nature of Things* (Rutgers University Press, New Brunswick, 1950), Bk. VI, verse 40. Brown says (p. xv): "This [new] translation is made in the belief that the meaning of Lucretius will be most intelligibly expressed if he is rendered, not in prose, nor into any of the more commonly used meters of English poetry, but into rhythmical periods which throw across to us line by line, as nearly as may be, the corresponding poetical periods of the Latin text. It is an attempt to present the subject matter of the poem to the English reader in a framework [hexameter verse] similar to that wherein Lucretius himself chose to convey the content of his mind to his own countrymen."

[18] It is significant that of the few really monumental works of creative literature, three have a philosophic function involving physical theories: *De Rerum Natura*, Dante's *Divina Commedia*, and Milton's *Paradise Lost*. For further discussions of science and poetry see, for example, F. S. C. Northrop, *The Logic of the Sciences and the Humanities* (Macmillan Company, New York, 1947), chap. IX, "The functions and future of poetry"; K. Mac-

Other Early Hypotheses

The essence of the Epicurean explanation, as set forth by Lucretius, was that particles streaming forth from the loadstone displace the air between it and the iron, and into the resulting partial vacuum, the iron moves. This hypothesis was restated in a slightly different form by Plutarch (A. D. 1st century):

> [The loadstone] emits strong exhalations, by which the adjoining air being impelled forceth that which is before it; and this [air], being carried round in the circle and returning into the vacuated place, forcibly draws the iron in the same direction.[19]

Plutarch's account extended this explanation so as to include the amber effect:

> In amber there is a flammeous and spirituous nature, and this by rubbing on the surface is emitted by recluse passages, and does the same that the loadstone does.[19]

Plutarch also pointed out a striking difference between the effects of the loadstone and amber: only iron will move to the loadstone, but a variety of small objects will move to rubbed amber. However, we shall see that his emphasis on this difference was not sufficient to reduce the prevalent confusion of the two effects.

That the early Chinese had knowledge of both the amber effect and the loadstone has been frequently asserted. Yet what is believed to be the first Chinese reference to the subject appeared in a "Eulogy of the magnet," by Kuo P'o (A. D. 4th century):

> The magnet draws the iron, and the amber attracts mustard seed. There is a breath which penetrates secretly and with velocity, and which communicates itself imperceptably to that which corresponds to it in the other object. It is an inexplicable thing.[20]

Apparently there is nothing in this Chinese reference that was not already known in the Western world.

A factual account of much of this early Western knowledge appears in *De Civitate Dei* of Saint Augustine (A. D. 5th century), the great philosopher of the early Christian church. Characteristic of his writings is the care with which he distinguished between verifiable and unverifiable secular knowledge:

> For my own part, I do not wish all the marvels I have cited to be rashly accepted, for I do not myself believe them implicitly, save those that either have come under my observation or anyone can readily verify, such as . . . the magnet, which by its mysterious and insensible suction attracts the iron but has no effect on straw.[21]

Thus, like Plutarch before him, Augustine noted that a magnet moves iron, but not straws. He also described, as had Plato, how a number of iron rings may be suspended in a chain from a loadstone as if the magnet "had communicated its own property to the iron it attracted, and had made it a substance like itself." It will be remembered that Lucretius[17] had described the effect of a loadstone on bits of iron in a brass bowl. To this Augustine added something new in telling how it had been reliably reported to him that a bit of iron on a silver plate would follow immediately and precisely a magnet moved around underneath the plate and without the intervening silver being affected at all.

Without pursuing such matters further, Augustine proceeded instead to make use of them for his larger purposes. If such common phenomena are inexplicable, he asked, why should it be demanded of man that he explain miracles by human reason? Thus, while Augustine was among the last of the early church fathers to be well versed in Greek philosophy and science, his dominant concern naturally was with religious and moral matters. Such matters indeed were

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Lean, *John Locke and English Literature of the Eighteenth Century* (Yale University Press, New Haven, 1936); D. Bush, *Science and English Poetry* (Oxford University Press, London, 1950). For a study of the politico-social and philosophic aspects of *De Rerum Natura*, see, for example, B. Farrington, *Science and Politics in the Ancient World* (George Allen & Unwin, 1939), chap. 14.

[19] Plutarch, *Platonic Questions*, translated by R. Brown, in *Plutarch's Morals* (Little, Brown, and Company, Boston, 1871), Vol. V, p. 436.

[20] Quoted by Park Benjamin, *The Intellectual Rise of Electricity* (D. Appleton and Company, New York, 1895), p. 74. This 595-page history covers the periods from antiquity through the work of Benjamin Franklin. Although naturally in need of revision in the light of historical researches and better understanding of electricity gained since 1895, the book has had an important, evident influence on many subsequent writers on electrical history. We have leaned heavily upon it in the preparation of the present article.

[21] Aurelius Augustinus, *De Civitate Dei*, translated by Dods (Edinburgh, 1871), Bk. XXI, Secs. 4, 6, 7; also by J. Healey (John Grant, Edinburgh, 1909). This great treatise is referred to as "the first Christian philosophy of history" by George Sarton in his *Introduction to the History of Science* (Williams & Wilkins, Baltimore, 1927–1948), Vol. I, p. 383; Sarton's history is the most scholarly, accurate and complete work available on the period from Homer to the end of the 14th century.

to absorb increasingly the attention of the scholars of Western Christendom for many centuries to come; the pursuit of mundane affairs and secular learning seemed, by comparison, to hold little that was of interest or commendable.

THE MEDIEVAL PERIOD

Although nothing new seems to have been added to the knowledge of the amber effect until the 12th century was reached, there were certain trends in the intervening period that deserve emphasis because of their relation to the sciences as they subsequently developed in the West. These trends were connected with activities in the monasteries of Western Europe, from about the 6th century onward and especially in the church schools instituted by Charlemagne near the end of the 8th century. There, secular philosophy and science were indeed not encouraged. Yet the early church scholars and schoolmen appear to have performed a notable service for the secular science of the future by fostering such traits and conceptions as respect for law and order in all things, habits of clear and logical thought, and of intellectual honesty, and the ability and willingness to think in general terms. These traits and conceptions, and others similar to them, are at once characteristic of Hellenic culture, the Hebraic-Christian tradition, and modern Western science—in other words, of the main elements out of which our Western civilization has been compounded. They doubtless helped to prepare Western minds to capitalize on Greek knowledge, once it was made available to the West, in the 12th century, and thus eventually to bring about the Scientific Revolution.[22]

The recovered Greek works, especially those of Aristotle, revealed a knowledge and understanding of natural phenomena incomparably greater than anything of the kind known to the medieval scholars. Attempts to disregard and even place bans on this secular, "pagan" knowledge were soon seen to be futile, and to reconcile it with religious knowledge therefore became a major problem. This difficult problem was solved for its day when a most astounding synthesis of the Aristotelian physics and philosophy with religious teachings was worked out in the 13th century by Thomas Aquinas, a scholar of the first rank and the greatest church philosopher after Augustine. The portion of Aristotelian science with which Aquinas was best acquainted and most concerned was the cosmology and dynamics; but he also had some acquaintance with other physical knowledge, including magnetism and the amber effect. To explain the amber effect, he adopted the hypothesis of Plutarch, according to which the surrounding air takes part in the phenomenon. There is a suggestion in his writings of the notion that a magnet not only attracts iron, but that at the same time the iron attracts the magnet. This notion of mutual attraction was not similarly applied to the amber effect, however; that the amber's attraction was purely one-sided was to remain the generally accepted belief for some four centuries to come.

Alexander Neckam

By the 12th century, in addition to amber and "lyncurium," a third substance, jet, was being mentioned as having the property of attracting when rubbed. According to Alexander Neckam,

> If you ask its value as an ornament, jet is black and brilliant; if its nature, water burns it and it is extinguished by oil; if its power, being heated by rubbing, it holds things applied to it, like amber; if its use, it is an excellent remedy for dropsy.[23]

Neckam, who was the foster brother of Richard I, is the earliest Englishman to be mentioned here. An able schoolman and student of nature, he was educated in an English school and then in Paris, where at the age of 23 he became a "distinguished lecturer" in the University. A few years later he returned to his native land.

In his factual descriptions of phenomena, Neckam seems to have relied to a considerable extent on his own observations, rather than following the common practice of the time of

[22] For an enlightening interpretation of general trends in the medieval period, see Whitehead, reference 2, chap. 1.

[23] Alexandri Neckam, *De Naturis Rerum . . .* (*c* 1180), ed., in Latin, by Thomas Wright (Longman, Green, Longman, Roberts, and Green, London, 1863), chap. XCVII, p. 181. For a comprehensive account of Neckam and his views, see pp. ix–lxviii of Wright's preface, which is in English. Wright's account is tempered somewhat with a modern evaluation by Lynn Thorndike, *A History of Magic and Experimental Science* (Macmillan Company, New York, 1923), Vol. II, chap. 43, pp. 188–204.

Girolamo Fracastoro (*c* 1478–1553). [From *Profili biobibliografici di medici e naturalisti celebri Italiani dal sec. XV° al sec. XVIII°*, Vol. 1 (4th thousand, corrected and amplified, Rome, 1932), Plate VIII facing p. 40.]

merely quoting from earlier authors. But in seeking to explain attractions and repulsions, he adopted a doctrine of "similitudes" and "dissimilitudes" much like one that certain Arabian writers had earlier proposed for the magnet. It was not far different from the ancient Greek doctrine of sympathies and antipathies.[16]

In Neckam's time it was still thought that, while chaff moves toward rubbed amber or jet, the latter will not move toward the chaff; and while iron moves to a loadstone, the latter will not move toward the iron. To explain all this, Neckham adopted the assumption that an attracting object acts more strongly than does the attracted object. If the two objects were to act on each other with equal strength, they presumably would counterbalance, and neither one would move. This also seemed to account for another still current belief, namely, that two loadstones had no manifest effect on each other; having equal degrees of similitude, the two stones would draw equally on each other and no movement supposedly would result.

Neckham gave what may be the earliest extant description of the mariner's compass and the procedure for using it,[23] and he indicated that the device was already in fairly wide use as an aid to navigation. By experimenting with it, he could easily have seen that the magnetized needle will turn toward iron, and if two similar needles are placed close together, both will rotate. These facts probably were already well known to many mariners, since the compass is not reliable as a direction-finder unless kept away from other magnetic materials. But the mariners were concerned with practical and immediate needs, probably had reasons for keeping knowledge of the compass as a trade secret and, moreover, had little intercourse with scholars or access to their abstruse writings. The scholars, on the other hand, seemed to be interested mainly in showing that any new phenomenon encountered was really not novel but could be reconciled with more familiar experiences and used to support some broad conceptual scheme. Neckam himself said that the purpose of his book was not primarily scientific or philosophic; rather it was to afford moral instruction, his descriptions of natural phenomena leading in each instance to some moral application or spiritual analogy. Until such specific phenomena as those of the amber effect and magnetism became matters of primary interest to curious scholars, there was little seeking for new facts by resorting to working hypotheses that could be tested experimentally. Until that time, new facts were likely to be encountered only by craftmen and workmen, and then only if the phenomenon concerned had already been found to have utilitarian value.

THE 15TH AND 16TH CENTURIES

Marsilio Ficino

That a magnet will move iron but not straw had been emphasized in antiquity by Plutarch and by Augustine. Now in the 15th century another important difference between the magnet and amber was noted, by Marsilio Ficino. Ficino was an Italian philosopher and writer and the earliest leader of the new school of Platonists in the Renaissance; as with the other humanists of the period "the whole of antiquity seemed precious in his eyes." What Ficino reported was that rubbed amber, unlike the magnetic needle, does not point in a north-south direction. He was also a student of astrology, and his explana-

tion of the difference was that the amber and the magnet were controlled by different stars. How he learned that rubbed amber does not exhibit directionality, he did not bother to say. If it had been with the help of a piece of amber mounted on a pivot, this would have marked the invention and use of the first electrical instrument. According to the available records this important experimental device was not invented until more than a century later.

Gerolamo Fracastoro

Not until the middle of the 16th century do we find a fourth substance added to the list of those showing the amber effect. This was the diamond, added by Gerolamo Fracastoro, or Fracastorio, an Italian physician and poet. Fracastoro, who is often called "the father of epidemiology," had studied and taught at Padua, one of the most progressive and tolerant of the 16th century universities. Seeking an explanation for the few facts known about the amber effect, he noticed how little similarity there is between the attracting substances—amber, jet, diamond —and the few things known in his time to be attracted—notably hairs and twigs. From this dissimilarity he inferred that

> . . . perhaps hairs and twigs are drawn to amber and diamond not because they are hairs, but because there is imprisoned within them either air or some other principle that is first attracted and that has reference and analogy to that which of itself attracts; and herein amber and diamond are as one, in virtue of a principle common to both.[24]

In other words, the attraction conceivably is not between the objects themselves, but between some "principle" in any of the attracting objects and the same "principle" in any of the objects attracted.

From this explanation there is deducible a working hypothesis that Fracastoro could have tested experimentally, even with the limited means at his disposal. It it were true that all attracting and attracted substances belong to the same species, in that all contain the same "principle," then conceivably any member of the species when rubbed would attract any other member. More specifically, it might have been found that a thread will move toward a rubbed straw or thread or even that the two will move toward each other. But Fracastoro did not mention such experiments. His explanation had served its purpose by accounting, to his satisfaction, for the few facts already known to him.

Fracastoro thought his explanation also applicable to magnets: attraction would occur when the "principles" present in the magnetic substances were analogous; repulsion, when they were contrary. Perhaps this extension of the doctrine represented nothing more than mere uncritical acceptance by Fracastoro of the still prevalent opinion that the amber and magnetic effects were identical or, at least, occurred for the same reason; for the known magnetic substances were not markedly varied or dissimilar, yet these were the characteristics that, in the case of the amber effect, had led Fracastoro to formulate his doctrine. However, he reported having seen silver attracted by a loadstone, and this would increase the number of substances that he thought to be magnetic. It is possible that the attracted object was a silver coin containing some iron.

Georgius Agricola

A second 16th century investigator who enters briefly into this history is Georgius Agricola, a German, the name Agricola being the Latinized form of the German surname Bauer. Agricola was a practicing physician who contributed to many fields of knowledge and eventually won fame as the founder of the science of mineralogy. In one of his books[25] he summarized the existing information on the properties and uses of amber. For instance, various alleged medical properties were mentioned, such as the supposed efficacy of the fumes from burning amber as a preventive of the dreadful plague. When amber is rubbed, he pointed out, it shows pronounced heating, which increases with the violence of the rubbing; and it draws to itself various small objects, including metal filings.

[24] Hieronymus Fracastorius, *Opera Omnia* (Venice, 1555), Bk. "De Sympathia et Antipathia," as quoted by William Gilbert, *De Magnete* . . . (London, 1600), translated by P. F. Mottelay (John Wiley and Sons, Inc., New York, 1893), p. 82.

[25] Georgius Agricola, *De Natura Fossilium* (Basel, 1546), Bk. IV. This edition consists of four different works bound together and with continuous pagination; Bk. IV of *De Natura Fossiluim* appears on pp. 237–247.

Jerome Cardan (1501–1576). [From *Profili Bio-bibliografici di medici e naturalisti celebri Italiani dal sec. XV° al sec. XVIII°*, Vol. 2 (3rd thousand, Rome, 1928), Plate VI, facing p. 40.] As has been pointed out by Benjamin (reference 20, p. 243), the statement of Cardan's age as 49 years is not in accord with the date of his birth as given by his biographers.

He saw no difference between amber's attraction and that of the magnet. The magnet's properties were recognized as less temporary, less illusory and frequently more pronounced. But these seemed to be differences in degree and not in kind, in quantity rather than quality.

Nevertheless, the uniqueness of some of the magnetic properties were becoming evident to those who were familiar with some systematic studies of magnetism that had been made as the mariner's compass came into increasingly wider use. Notably, those properties that made the compass useful in navigation were lacking in the amber effect, for which no practical use had been discovered. So it is not surprising to find an attempt made to clear up the confusion between the two effects, and this at about the time when Agricola and Fracastoro were lending emphasis to their similarity.

Jerome Cardan

This new approach appeared in a mid-sixteenth century treatise entitled *On Sublety* by Jerome Cardan, a celebrated Italian mathematician and physician.[26] Cardan's account begins with a summary of the various properties and uses of amber as they were given in Agricola's book. Then comes the unqualified statement: "the magnet stone and the amber do not attract in the same way." The two phenomena differ in quality and not merely in degree. What is more, Cardan listed his supporting evidence:

> The amber draws everything that is light; the magnet, iron only.
>
> The amber does not move chaff toward itself when something is interposed; the attraction of the magnet for iron is not similarly hindered.
>
> The amber is not mutually attracted by the chaff; the magnet is drawn by the iron.
>
> The amber does not attract at the end [does not exhibit polarity]; the magnet attracts the iron sometimes at the North and sometimes at the South.
>
> The attraction of the amber is greatly aided by heat and friction; that of the magnet, by cleaning the attracting part [thus removing foreign matter and scale].

Included in these empirical generalizations are two important assertions about the amber effect that we have not encountered in any earlier writings, namely that (1) *all* small objects will move toward the amber; (2) no movement will occur when there is an interposing barrier. That the barrier—a piece of cloth hung between the rubbed amber and light object is often cited in later writings—does not destroy but merely weakens the attraction, was noted only a few decades after Cardan. His statement that the magnet's attraction is not weakened by an intervening barrier appears to be the first record of this phenomenon since Augustine's comment 11 centuries earlier that iron lying on a silver plate can be moved by a magnet held under the plate. Shortly after Cardan's time it was shown that a barrier of iron will cut off the magnet's attraction.

To account for the five general differences that he had noted, Cardan devised what may well be the first hypotheses pertaining to the amber effect as a phenomenon distinct from magnetism. He assumed that the rubbed amber emits a "fatty and glutinous humor," and that chaff or

[26] Hieronymi Cardani, *De Subtilitate* . . . (1551), Bk. v; see *Hieronymi Cardani* . . . *Opera Omnia* . . . (Lyons, 1663), Vol. 3, Bk. v, pp. 443–444. For biographies in English see: R. B. Lindsay, Am. J. Phys. 16, 311 (1948), an excellent, brief account; Jerome Cardan, *The Book of My Life* (*De Vita Propria Liber*), translated by Jean Stoner (E. P. Dutton and Company, Inc., New York, 1930), which for its clearness and frankness of self-revelation "stands almost alone among records of its class"; W. G. Waters, *Jerome Cardan, a Biographical Study* (Lawrence and Bullen, London, 1898).

other dry objects, while absorbing this "humor," move toward the amber. For, he supposed, "every dry thing, as soon as it begins to absorb moisture, is moved toward the moist source, like fire to its pasture."

But Cardan had also noted that the hotter the amber, the more it draws. Apparently he felt that this supposed increase in drawing strength with heating was not sufficiently explained by the hypothesis of "the fatty and glutinous humor." So he invoked another ancient notion to the effect that attraction can result from the "force of fire"; and to this he added his own opinion that the amber effect might therefore be likened to the action of the cupping glass, a device that physicians had been employing since antiquity to bleed patients and also to relieve internal congestions. In use, the cupping glass was first warmed so as to drive out some of the air, and then its mouth was pressed against the skin. As the cup cooled, the pressure of the air in it decreased and blood was driven to the surface of the skin, which may or may not have been opened. However, this is the modern explanation of the action of the device and was not known to Cardan, in whose day little was understood about the effects of air pressure. He thought it was a supposed "force of fire" that drew the blood to the skin in the cupping glass and, analogously, drew light objects to amber. Noticeably this "force of fire" explanation is not consistent with his other hypothesis attributing the attraction to one of moist bodies for those that are dry.

Cardan's *On Subtlety* gained wide popularity, dealing as it did with those "subtle" phenomena that are "sensible by the senses or intelligible by the intellect, but with difficulty comprehended." Since many earlier writers had unhesitatedly tried to explain not only phenomena that were difficult to comprehend, but those not comprehended at all, Cardan's ambitious program seems by comparison very modest indeed. About one-third of *On Subtlety* deals with physical matters, and the remainder runs the gamut "from plants and animals through man to God and the universe, with some respects paid along the way to dreams, objects of art and demons." One finds in the book much that is now shear fantasy, and also many contradictions; but mixed with these are sound physical learning and a most advanced spirit of speculation.[27] One cannot be sure whether Cardan obtained some of his facts about the amber effect by deducing from his broad doctrines, more specific working hypothesis, or theorems, and then testing the latter experimentally; or whether, instead, he had all the facts to begin with, and then framed his broad hypotheses, or doctrines, in the light of them. In his book he stated the broad hypotheses first, and then listed the facts as supporting evidence. He gave no details of any experiments. But, whatever the case may be, his broad *ad hoc* hypothesis of a "fatty and glutinous humor" cannot be dismissed as being merely absurd. The facts known to him indicated that the amber attraction was due to something of a material character that ordinarily was imprisoned in the amber but could be liberated by rubbing. The rubbed amber has no visible connection with the small objects moving toward it; and, according to the Aristotelian dynamics, to which Cardan largely subscribed, one object could not exert force on another object unless the two were either in contact or connected by some other material body. For Cardan the notion of "action at a distance" would have been totally unacceptable; it would have been regarded as a reversion to the sort of mysticism and magic from which some of the scientist-scholars of the time were trying to escape.

Indeed, the notion of an invisible "something" proceeding out of the rubbed amber by no means ended with Cardan. It came to be called an "effluvium" or "electrical fluid"; and, as additional knowledge of the amber effect was acquired, various new properties were ascribed to this effluvium and different opinions were expressed as to just how it acted on small objects so as to move them toward the amber or other electrified body. This was the electrical doctrine that prevailed until Franklin's time—the middle of the 18th century. Only then was the conception of "action at a distance" extended to electrical phenomena, as it had been applied earlier to gravitation. Moreover, as E. T. Whittaker[28] has pointed out, the effluvium doctrine,

[27] Lindsay, reference 25.

[28] E. T. Whittaker, *A History of the Theories of Aether and Electricity* (Longmans, Green and Company, New York, 1910), p. 45; (revised ed., 1951), p. 48.

which "practically ended with Franklin, bears a suggestive resemblance to that which nearly a century later was introduced by Faraday; both explained electrical phenomena without introducing action at a distance, by supposing that something which forms an essential part of the electrified system is present at the spot where any electric action takes place; but in the older theory this something was identified with the electric fluid [effluvium] itself, while in the modern view it is identified with the state of stress in the aether." It was only in the interval between the fall of one school and the rise of the other that the theory of action at a distance was dominant.

In the period of more than 20 centuries that we have now covered, the accumulation of knowledge about the amber effect and its relation to other physical phenomena seems so small as to be almost negligible. As to the effects of this knowledge on the thinking of subsequent investigators, these were by no means negligible. Even in amount this early knowledge is small only in comparison with the tremendous developments in electrical science that were to occur in the four centuries to follow. Only by taking into account the preceding ages of man, which cover more than 99 percent of his whole existence, can we appreciate that the facts and speculations reviewed here, meager as they are, represented a substantial gain.

Noticeably several physicians contributed to this early knowledge, their initial interest in amber possibly having been aroused by its alleged medical values. But now, following Cardan, a problem from an entirely new field of natural knowledge—astronomy—coupled with interests in the earth's magnetism and the compass as a nautical instrument, was soon to lead another physician, William Gilbert, to the study of the amber effect. His great treatise, *De Magnete*, incorporates frequent mention, often with critical discussion, of the findings and opinions of his predecessors; and interpretations of many of his comments become easier if the reader has some acquaintance with this early knowledge such as is afforded by the present review.

Gilbert retained many notions and modes of thought that clearly are traceable to ancient, medieval and Renaissance writers, for their doctrines and notions still largely prevailed in his day, and from the climate of opinion in which one is working, no person can completely escape. Nevertheless, Gilbert's methods and spirit of attack turned out to differ amazingly from anything we have encountered in this prenatal history of electrical science.[29]

[29] This paper is intended both as a contribution to the literature on the early history of electricity and to provide background and lecture material for courses in which the evolution of physical concepts and methods is stressed. A shorter, modified version, more suitable as collateral reading for the average student, will appear in *The Development of the Concept of Electrical Charge*, scheduled for publication in 1954 as one of the *Harvard Case Histories in Experimental Science*.

Readers wanting a better acquaintance with the developments in magnetism before 1600 will find brief accounts in A. Wolf, *A History of Science, Technology and Philosophy in the 16th and 17th Centuries* (Macmillan Company, New York, first ed., 1935; revised by Douglas McKie, 1951), pp. 290–292, and in L. W. Taylor, *Physics, the Pioneer Science* (Houghton Mifflin Company, Boston, 1941), pp. 577–594. For a comprehensive account, see Benjamin, reference 20, pp. 1–257.

Summer Meeting of AAPT

The Summer Meeting of the American Association of Physics Teachers will be held in Pittsburgh, Pennsylvania, on June 25, 26, and 27, 1953. Arrangements have been made with the Webster Hall Hotel for room reservations for our members. It is important and necessary that members write to the hotel directly to receive confirmation of their reservations, mentioning their connection with the Association.

Most of the meetings will be held in the air-conditioned rooms of the nearby Mellon Institute. Several symposiums are planned, and active local preparations are being made by the Western Pennsylvania, Central Pennsylvania, and Appalachian Sections in collaboration with PROFESSOR CHAS. WILLIAMSON, *Carnegie Institute of Technology*.

An Experiment on Cosmic Rays

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(Received September 29, 1952)

An experiment suitable for an undergraduate laboratory course is described. A two-counter cosmic-ray telescope is employed in conjunction with a Rossi-type coincidence circuit. Details of the circuits are given. The methods of calculating the intensities of the cosmic rays are discussed, and a typical set of experimental results is shown. These results are used to illustrate the statistics of the rate of arrival of cosmic-ray particles.

THE experiment to be described in this paper is designed to form an introduction to the technique of coincidence counting, and also to enable the statistics of rate of arrival of cosmic-ray particles to be studied. This experiment has been performed by students in the senior undergraduate laboratory at Sydney for the past few years, and has proved to be a very valuable addition to the practical course. Before commencing the experiment the student has completed a simple introductory experiment on Geiger counters and scaling circuits.

Figure 1 is a photograph of the equipment. The basic apparatus is a simple cosmic-ray telescope, consisting of two Geiger counters clamped parallel to one another in spring brass holders on a wooden board. The holders are movable to permit the counter separation to be varied, while the board can be rotated about a horizontal axis. A self-quenched type of counter filled with a mixture of argon and ethyl alcohol is used. The counters have cylindrical copper cathodes 12 cm long and 2 cm in diameter mounted in glass envelopes, and the center wires are surrounded by glass sleeves projecting 2 cm beyond the ends of the cathodes. The sensitive length of each counter is thus 8 cm. The dimensions of the counters are not in any way critical.

The Geiger counters require a negative voltage of from 1000 to 1200 volts for their operation. This voltage is obtained by using a low voltage rf oscillator, the output of which is stepped up by a transformer and then rectified. The circuit of the rf power supply is shown in Fig. 2. The three coils L_1, L_2, L_3, are pi-wound on the same former. The output voltage may be changed by altering the capacity C. A commercial coil assembly designed for use in a cathode-ray oscillograph is employed, but it is possible to wind satisfactory solenoidal coils in the laboratory.[1] The high tension supply to the rf oscillator is kept constant by means of a VR150 tube, and this maintains the output voltage constant to within about 30 volts. Since the counters have plateaus of about 150 volts, this regulation is quite adequate. The principal advantage of the rf supply over the conventional mains-frequency supply is that with the rf supply there is no fear of a dangerous electric shock.

A general purpose rectifier power pack is used to provide dc and heater voltages for the rf high voltage power supply and the coincidence counter. A small cathode-ray oscillograph using a type 902 tube is also operated from the same power pack, but a separate self-contained oscillograph would be more convenient.

THE COINCIDENCE CIRCUIT

A widely used coincidence circuit, due to Rossi,[2] is employed. The circuit diagram is shown in Fig. 3. The plates of the two 6J7 tubes are tied together, and since the dc resistances of the pentodes are only a few thousand ohms the common plate voltage will be quite low. If a negative pulse appears at one of the 6J7 grids that tube will be cut off, but the other tube will still be conducting and hence the plate voltage will not increase very much. However, if both tubes are cut off simultaneously the plate voltage will rise rapidly to its limit and a large positive output pulse will be produced. The discrimination, that is, the ratio of the change in plate voltage for two coincident pulses to the change

[1] C. J. Dickinson, Wireless World **57**, 70 (1951).
[2] See J. Strong, *Procedures in Experimental Physics* (Prentice-Hall, Inc., New York, 1944), pp. 289–291.

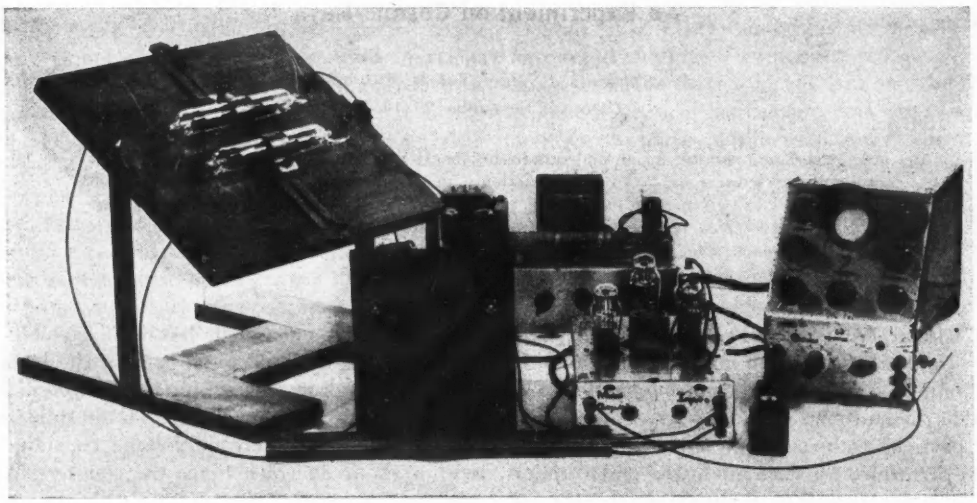

FIG. 1. Photograph of the equipment. From left to right—telescope, rf supply, coincidence circuit, recorder, oscillograph. The power pack is at the rear.

in plate voltage for single pulses, can be altered by varying the screen potential.

The pulse from the plates of the 6J7 tubes is used to trigger a conventional scale-of-two thyratron circuit. A mechanical recorder in the plate of one thyratron gives an audible click for every pulse, but the number on the recorder is advanced only for every second pulse. To obtain the number of pulses recorded, the number shown is multiplied by two, and one is added if the appropriate thyratron is seen to be conducting. It is found that a scale-of-two circuit is by reason of its symmetry much more stable in operation than a single valve counting circuit and has less effect on other apparatus connected to the same power pack.

The adjustment of the circuit is best done in the following way. The bias on the thyratron grids is set at the maximum negative value. The cylinder of one of the Geiger counters is connected to the 1200-volt negative supply and the center wire to *both* input terminals of the coincidence circuit. The output of this circuit is connected to the cathode-ray oscillograph. The voltage on the Geiger counter is adjusted so that the counter is near the middle of the plateau region, and the thyratron bias is reduced until the recorder operates satisfactorily. The lead to one of the input terminals is then disconnected; the pulses should now be so small that they do not operate the recorder.

The other Geiger counter is now adjusted to the middle of its plateau by connecting its center wire to both input terminals. Finally, the Geiger counters are set as close together as possible in the vertical plane and are connected to the separate input terminals of the coincidence circuit. Large amplitude pulses due to cosmic-ray particles passing through both counters should be observed on the oscillograph, superimposed on a background of small pulses due to particles passing through a single counter. Only the large pulses should trip the recorder.

It is advisable to keep the two leads from the Geiger counters to the coincident circuit as far apart as possible, otherwise capacity coupling between the leads may cause the pulses due to particles passing through a single counter to increase in magnitude.

NOTATION FOR THE SPECIFICATION OF COSMIC-RAY INTENSITIES

The following symbols may be defined:

$N(\theta)$ the number of coincidences per minute with the telescope at an angle θ to the vertical.

$N(0)$ the coincidence rate in the vertical direction.

$J(\theta)$ the intensity of cosmic rays arriving in the direction θ, i.e., the number of particles per cm² per unit solid angle per minute.

$J(0)$ the intensity of cosmic rays arriving in the vertical direction.

J_1 the omnidirectional particle intensity per unit horizontal area, i.e., the number of particles arriving per cm² per minute.

J_2 the omnidirectional particle intensity per unit sphere, i.e., the number of particles per minute which pass through a sphere of unit cross section (directly related to the ionization produced in an ionization chamber).

a the sensitive length of each counter.

b the inside diameter of each counter.

d the separation of the axes of the two counters.

The solid angle subtended by the top counter at any point on the cross section of the bottom counter is approximately ab/d^2, and the sensitive area of cross section of the bottom counter is ab. Hence $N(\theta)$ and $J(\theta)$ are connected by the following approximate equation:

$$N(\theta) \doteqdot J(\theta)(ab/d^2)ab,$$

or

$$N(\theta)/J(\theta) \doteqdot (ab/d)^2. \tag{1}$$

A more precise relation between $N(\theta)$ and $J(\theta)$ is[3]

$$\frac{N(\theta)}{J(\theta)} = \left\{\frac{b^2}{4}\left(3-\tan^{-1}\frac{a}{d}+\frac{a^2}{d^2+a^2}\right) + \frac{\pi b^3}{8d}\left(1-\frac{d^2}{d^2+a^2}\right)^2\right\}\frac{\beta}{\alpha}, \tag{2}$$

where

$$\sin\beta = \tan\alpha = (b/d).$$

The first term in the curly brackets on the right-hand side of Eq. (2) is obtained by considering the counters as rectangular sensitive areas ab. The second term is a correction due to the circular ends of the counters. The factor β/α arises from the circular cross-sectional area of the counters.

[3] K. Greisen, Phys. Rev. 61, 212 (1942).

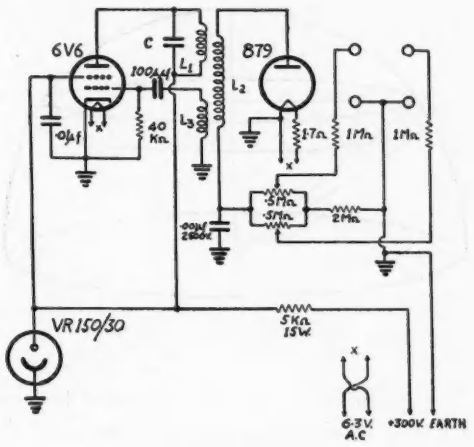

FIG. 2. The rf power supply.

In the counters used in the experiment, a is 8 cm, b is 2 cm, and d is 10 cm. On substitution of these values Eq. (1) becomes

$$J(\theta) = (1/2.56)N(\theta),$$

while the more accurate Eq. (2) gives

$$J(\theta) = (1/2.25)N(\theta).$$

Thus a fairly close approximation is obtained by using the simple expression for $J(\theta)$ given by Eq. (1).

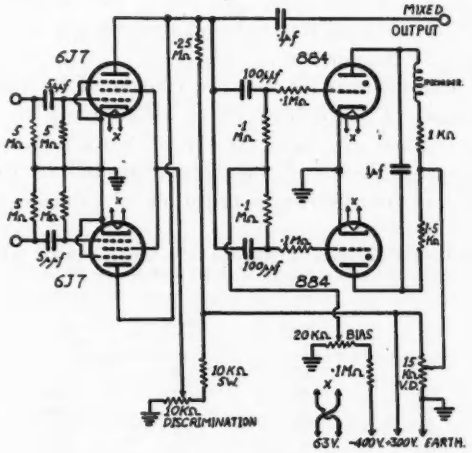

FIG. 3. The coincidence circuit.

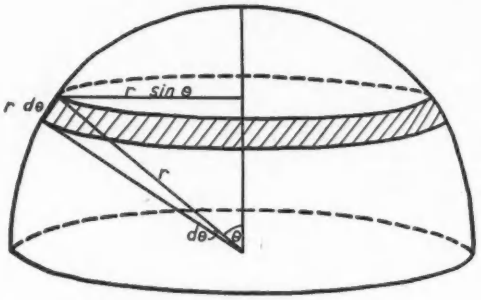

FIG. 4. Diagram illustrating the method of calculation of J_1, J_2.

The quantity J_1 may be calculated by considering a hemisphere of large radius r at the center of which is a horizontal unit area and summing the number of particles passing through the area in the direction θ over all values of θ (see Fig. 4). Thus

$$J_1=\int_0^{\pi/2} J(\theta)\cos\theta\frac{2\pi r\sin\theta r d\theta}{r^2}$$

$$=\frac{\pi}{2.25}\int_0^1 N(\theta)d(\cos^2\theta)$$

$$=\frac{\pi}{2.25}\times(\text{area under } N(\theta):\cos^2\theta \text{ graph}).$$

The factor $\cos\theta$ arises because in the definition of $J(\theta)$ the area referred to is that normal to the path of the particles.

The value of J_2 may be calculated by considering a large hemisphere as above, with a unit sphere placed at the center. Since the area of cross section of the sphere normal to the path of the particles is unity for all values of θ, it follows that

$$J_2=\int_0^{\pi/2} J(\theta)\frac{2\pi r\sin\theta r d\theta}{r^2}$$

$$=\frac{2\pi}{2.25}\int_0^1 N(\theta)d(\cos\theta)$$

$$=\frac{2\pi}{2.25}\times(\text{area under } N(\theta):\cos\theta \text{ graph}).$$

TABLE I. Coincidences recorded by a simple counter telescope at various angles with respect to the vertical.

θ	No. of particles	Period (min)	$N(\theta)$
0°	87	60	1.45
20°	40	30	1.33
40°	27	30	0.90
60°	11	30	0.37
80°	2	30	0.07

DETAILS OF EXPERIMENT

The counters are first placed 10 cm apart in the vertical plane and the number of coincidences occurring over a period of one hour is determined. The times at which the coincidences occur are noted to the nearest second from a stop watch or chronometer.

The telescope is now set in turn at angles of 20°, 40°, 60°, and 80°, to the vertical, and the number of coincidences over a period of half an hour determined for each of these angles.

A typical set of results, obtained in Sydney at altitude 25 meters, geomagnetic latitude 41°S, is shown in Table I.

When the experimental results have been obtained, graphs of $N(\theta):\cos^2\theta$ and $N(\theta):\cos\theta$ are drawn and the quantities $J(0)$, J_1, J_2, calculated. For the values given in Table I the graphs are shown in Fig. 5 and Fig. 6. It will be observed that the $N(\theta):\cos^2\theta$ graph is a straight line.

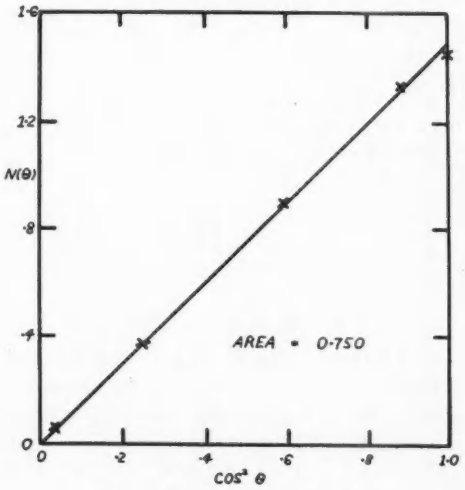

FIG. 5. The $N(\theta):\cos^2\theta$ curve.

In the present case the following numerical values are obtained:

Particles arriving in the vertical direction, $J(0)=0.64$ particle per minute per unit solid angle per cm²;
Particles crossing a horizontal area, $J_1=1.05$ particles per minute per cm²;
Particles crossing a sphere, $J_2=1.45$ particles per minute per cm² cross section.

These results compare favorably with the corrected values of Greisen,[4] which are as follows (for an altitude of 259 meters):

$J(0)=0.73$ particle per minute per unit solid angle per cm²,
$J_1=1.15$ particles per minute per cm²,
$J_2=1.54$ particles per minute per cm² cross section.

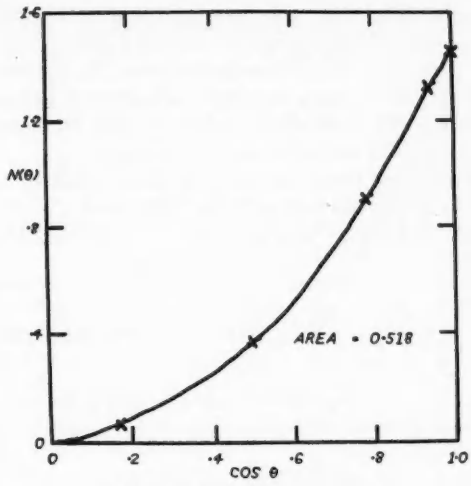

FIG. 6. The $N(\theta)$:cosθ curve.

STATISTICS OF THE COUNTING OF COSMIC-RAY PARTICLES

The measurements of the times at which particles pass through the cosmic-ray telescope in the vertical direction enable the student to check whether the rate of arrival follows the theoretical Poisson distribution.

If μ is the average number of particles arriving per minute, the probability $P_n(t)$ of exactly n particles arriving in t minutes is given by

$$P_n(t)=e^{-\mu t}\frac{(\mu t)^n}{n!} \qquad (3)$$

Taking as an example the experimental values listed in Table I, 87 particles were recorded in the hour. Hence in this case the value $87/60=1.45$ will provide an estimate of μ. Table II shows the number of minutes in the hour for which 0, 1, 2, ···, particles were observed, together with the predicted numbers of minutes as derived from Eq. (3). To check the agreement between the observed and predicted numbers, the student may apply a χ^2 test.[5] Grouping the last two classes together, a value 0.304 is obtained for χ^2 with 3 degrees of freedom. From the χ^2 tables the corresponding P, the probability that χ^2 will exceed the observed value, is 0.96. Hence the agreement in the present case is better than would be obtained in most experiments of this type.

TABLE II. Number of minutes in which n particles arrived.

n	Observed No.	Predicted No.
0	13	14.1
1	22	20.4
2	14	14.8
3	7	7.2
4	4	2.6
>4	0	1.0

The statistics of the intervals between the times of arrival of successive particles is also of interest. If a particle arrives at the instant $t=0$, the probability that no further particles will arrive before a later time t_1 is $P_0(t_1)$. Hence the

TABLE III. Intervals between the arrival of successive particles.

Interval Range (sec)	No. Observed	No. Predicted
0–9	15	17.6
10–19	17	14.7
20–29	13	11.5
30–39	8	9.0
40–49	6	7.2
50–59	4	5.5
60–69	3	4.4
70–79	2	3.4
80–89	6	2.8
90–99	4	2.1
>99	8	7.7

[4] Reference 3; see Phys. Rev. 62, 323 (1942) for a correction to the values given in this paper.

[5] R. A. Fisher, *Statistical Methods for Research Workers* (Oliver and Boyd, Edinburgh, 1948), pp. 83–84.

probability that the next particle will arrive at a time t in the range $t_1<t<t_2$ is $P_0(t_1)-P_0(t_2)$.

The observed intervals between the arrivals of successive particles in the vertical direction are distributed as shown in Table III. The predicted distribution is also shown in this table.

With the longer intervals grouped together as indicated at the left in Table III, a value 3.76 is obtained for χ^2 with 6 degrees of freedom. The corresponding value of P is 0.71, and so the agreement between predicted and observed values is good.

ACKNOWLEDGMENT

We should like to thank Mr. S. W. Hutchinson for his assistance in the construction of the equipment and in the preparation of the photograph and circuit diagrams for this paper.

Trend Analysis of Physical Data

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The problem of reduction of data, i.e., the derivation of the value of certain physically meaningful parameters which enter into the analytic description of the trend of the data, is solved by a generalization of the usual methods of fitting empirical curves. In taking a rather general point of view toward this method of trend analysis, the problem of "closeness of fit" is formulated in a new way and investigated briefly. Some applications of the method presented herein are to the problems of the analysis of radioactive samples, of the responses of electrical networks or electromechanical systems, of instrument records and vibrations including drift of the baseline of recording instruments, of variable illumination from stars, and of statistical distributions, and to the problem of differentiation of empirical data.

I. AN ILLUSTRATIVE EXAMPLE

THE problem of reduction of data arises in all experimental investigations in which numerical data are gathered. For example, let us suppose that we are given a certain set of data representing the number $y(t)$ of particles of a specified kind given off by some sample of radioactive material up to time t. Let us further suppose that it is suspected, either on the basis of these data, or from other information, that the source is composed of, say, two kinds of radioactive atoms, each yielding particles of the specified kind, but present in unknown proportions and with unknown individual lifetimes (or decay constants). The theory of radioactive disintegration indicates that the data should be representable approximately in the functional form

$$\eta(t)=N_1(1-e^{-\lambda_1 t})+N_2(1-e^{-\lambda_2 t}), \qquad (1)$$

where N_1 and N_2 are, respectively, the numbers of atoms of each kind present when $t=0$, and λ_1 and λ_2 are their respective decay constants. The problem of analyzing the data is that of determining the values of N_1, N_2, λ_1, and λ_2.

There have been given methods for accomplishing this, of course,[1] but these are sometimes subject to certain disadvantages, which it is not our purpose to dwell upon here. One of the difficulties in the determination is evidently that the decay parameters enter transcendentally in the expression (1). If there were more than two components, or if the required functional form involved damped sinusoidal terms with undetermined frequencies, the difficulties would evidently be compounded.

It is our purpose to present here a uniform method for finding the unknown parameters applicable not only in the special radioactive decay case described in the foregoing, but to the suggested more complicated cases as well, and even to more general situations which might arise. However, before we outline the general method and describe some of its applications and ramifications, we shall complete the discussion of the special example. In this way the chief idea underlying the method should become clear.

[1] A. S. Householder, "Analyzing exponential decay curves," Proc. Sem. Sci. Computation (November, 1949).

If we tried to fit the data straightforwardly by least squares, say, making the quantity

$$\langle (y-\eta)^2 \rangle \tag{2}$$

a minimum,[2] by varying N_1, N_2, λ_1, and λ_2, then the problem of determination of these parameters would be exceedingly complicated. But now note that the expression $\eta(t)$ in Eq. (1) satisfies an integral equation of the form

$$\eta(t) = a_1 \int_0^t \eta dt + a_2 \int_0^t \int \eta dt^2 + a_3 t + a_4 \frac{t^2}{2}, \tag{3}$$

in which a_1, a_2, a_3, and a_4 are constants.

This is most easily shown formally by deriving the differential equation which η satisfies, namely,

$$\ddot{\eta} + (\lambda_1+\lambda_2)\dot{\eta} + \lambda_1\lambda_2\eta = \lambda_1\lambda_2(N_1+N_2) \tag{3a}$$

and then integrating it twice, utilizing the initial condition

$$\eta(0) = 0, \quad \dot{\eta}(0) = N_1\lambda_1 + N_2\lambda_2. \tag{3b}$$

Alternatively one might have inferred the form Eq. (3) directly from a knowledge of the physical system.

In any event, we wish to point out that the use of *integral* relations such as Eq. (3), instead of the equivalent *differential* system Eqs. (3a) and (3b), is a main feature underlying our method and has certain advantages.

First, the constants a_1, a_2, a_3, and a_4 appear *linearly* in a single relation (3). Second, all the parameters specifying the system and the initial conditions occur in a very convenient way in terms of the a's. Third, the indicated integral operations are far more easily and accurately carried out in a practical case than would be difference or differential operations. Finally, the range of the integration operations need not be an infinite one. These advantages are not possessed by some other formulations we have considered.

The linearity of Eq. (3) suggests that instead of minimizing Eq. (2), we either minimize with respect to the a's,[3]

$$\left\langle \left(y - a_1 \int_0^t y dt - a_2 \int_0^t \int y dt^2 - a_3 t - a_4 \frac{t^2}{2} \right)^2 \right\rangle, \tag{4}$$

or determine the a's by setting the expression in brackets in Eq. (4) equal to zero at four judiciously chosen points. In any event, we are led to *linear* simultaneous equations for the determination of the a's.[4] Then, if we assume that Eq. (1) would be an exact description of the decay process if there were no statistical fluctuations or errors in the data, we must solve the integral equation (3) to determine the constants N_1, N_2, λ_1, and λ_2. But this latter is most easily accomplished by noting that λ_1 and λ_2 are the roots of the equation

$$\lambda^2 + a_1\lambda - a_2 = 0, \tag{5}$$

whereas N_1 and N_2 are found from

$$N_1+N_2 = -a_4/a_2, \quad N_1\lambda_1 = N_2\lambda_2 = a_3. \tag{6}$$

(The case in which a_2, and hence a_4, vanishes means that one component is inert, since its decay constant λ_2, say, is zero; the amount present N_2 is indeterminate from such a radioactivity observation, although N_1 and λ_1 are easily found.)

The generalization of this procedure to any number of radioactive components is immediate, and the solution of the secular and linear equations for the corresponding N's and λ's is conveniently found by the use of Newton's method of division.[5] It is possible, however, to take an even more general point of view of the procedure. This we describe briefly in the following section.

[2] In case the data are given in such a form that graphical or numerical integration is feasible, this bracket notation $\langle\ \rangle$ could mean integration of the enclosed quantity over the relevant time interval of observation. Or if the data are given numerically at specified instants, the bracket notation might signify summation of the enclosed quantity over these instants. This is a matter of taste and the particular situation encountered.

[3] Here we suppose $\int_0^t y dt$ means either integration or summation of the data y up to time t, whichever is appropriate.

[4] If we let

$$\alpha_1 = \int_0^t y dt, \qquad \alpha_3 = t$$
$$\alpha_2 = \int_0^t \int y dt^2, \quad \alpha_4 = \frac{t^2}{2}$$

then in the case of the minimization procedure these equations are

$$\langle \alpha_i y \rangle = \sum_{j=1}^{4} a_j \langle \alpha_i \alpha_j \rangle \quad (i = 1, \cdots, 4).$$

[5] G. E. Hudson, J. Math. Phys. **29**, 52 (1950).

II. DESCRIPTION OF THE GENERAL METHOD

Let there be given, ordinarily as the result of some set of measurements, a set of data representing a functional correspondence between two variables used in the description of the state of a physical system.[6] One of these we shall denote as the dependent variable y (frequently a generalized coordinate) and the other, the independent variable t (often the time). The data may be given in tabular form, or graphically, or in some special cases, analytically, on the interval of observation $(0, T)$. We further assume the possibility of executing in a practical manner various numerical or graphical operations on the data, to be introduced subsequently.

Such data, in part, represent the "response" of a physical system to some known type of external influence or "stimulus";[7] the remainder of the data, or "residue," may be attributed to a "complementary stimulus" about whose characteristics only statistical information of some kind may be known. In the special case described in Sec. I, the data $y(t)$ were composed of the response $\eta(t)$, accurately given by Eq. (1), and a residue, represented by the difference between y and η and resulting from statistical fluctuations in the decay rate, and errors in observation. The stimulus depended in this case on the initial amount of radioactive material present. We may consider that it was specified completely by the terms a_3t and $\frac{1}{2}a_4t^2$ in the Eq. (3), since if a_3 and a_4 were to vanish then there would evidently have been no radioactive material present (and hence no data). If we had considered the complementary stimulus, it would have been measured by the expression inside the parentheses in Eq. (4).

It also proves convenient for us sometimes to refer to a system and stimulus as a unit which might be termed the "generator" of a response; in our special case, the system was described in terms of the decay constants of the atoms; a more complete analytical specification was given by the terms η, $a_1\eta dt$, and $a_2\eta dt^2$ in Eq. (3). Hence the *type* of generator we dealt with in the example was specified by the functions and functionals of the time and response,

$$\eta, \int_0^t \eta dt, \int_0^t\int \eta dt^2, t, \frac{t^2}{2}.$$

The particular generator which yielded the response part of the data was chosen by finding specific values for the parameters a_1, a_2, a_3, and a_4, after postulating some criterion that the complementary stimulus had to satisfy. It is clear then that there must be given or assumed, in addition to the data, information concerning the class to which the generator belongs and information concerning the complementary stimulus.

Then, for our purposes, the problem of reduction of data may be posed as follows: (A) How may one deduce definite values of the parameters which uniquely specify the generator? (B) How may one derive from the data the response of the system to the stimulus and hence separate the residue from the response? (C) How may one determine the complementary stimulus?

It is the purpose of this paper to specify one method for analyzing uniformly the data collected from any of a large variety of physical generators. This method has been indicated in the example in Sec. I; in general, it may be described as follows:

From the given information concerning the system and stimulus, we infer to what generator class they belong. That is, we choose a set of functional operators $F_i(\eta; t)$ (in general, also functions of the independent variable) so that the response of any member of the generator class, including the specific one in question, is completely determined as the solution $\eta(t)$ of the equation

$$F_0(\eta; t) = \sum_{i=1}^{n} a_i F_i(\eta; t). \tag{7}$$

For each generator, specified by particular values of the a's, one may define from the data y the quantity

$$\gamma(t; a_1 \cdots a_n) = F_0(y; t) - \sum_{i=1}^{n} a_i F_i(y; t). \tag{8}$$

It is clear that we may regard γ as that stimulus

[6] We restrict ourselves in this paper to a discussion of two variables only. It is possible to generalize to more than two.

[7] J. D. Trimmer, *Response of Physical Systems* (John Wiley and Sons, Inc., New York, 1950). Trimmer prefers the term "forcing." In this paper we include initial and boundary conditions as stimuli.

which, if appended to a given generator, would yield the data $y(t)$ exactly as response. Hence we call it the complementary stimulus.

We now apply our assumed or given information concerning the complementary stimulus. It may be that from physical considerations we can infer that the complementary stimulus for that generator which actually leads to the observed data is zero at n instants. Fixing γ in this way, we are led to n linear simultaneous equations to determine the n parameters a_i, specifying a particular generator. A disadvantage of such collocation methods is that they sometimes weigh the data improperly, even to the point of excluding some of the pertinent information. The next simplest method (which includes the former as a special case) is a method of moments. Certain suspected statistical properties of γ may lead to the conclusion that the numbers

$$\langle \gamma w_j(t) \rangle,$$

where the $w_j(t)$ are known weighting functions, for that generator which actually leads to the observed data may have specific values M_i, $(i=1, \cdots, n)$. Thus again we are led to linear simultaneous equations for the a_i's.[8] Finally we mention that it may be known that γ, for the set of a_i specifying that generator which produced the observed data, approximates some given function $g(t)$ in the sense, for example, of least squares. Then it is clear that the a_i would be determined from the linear simultaneous equations

$$\sum_{i=1}^{n} a_i \langle F_i F_j \rangle = \langle (F_0 - g) F_j \rangle \quad (j=1, \cdots, n). \quad (9)$$

In each of these cases knowledge concerning γ led to a determination of the a_i, that is, of the system and stimulus.

After solving Eq. (7), with these values of the a_i, to determine the response η (usually desired in analytic form), the residue $\phi(t)$ may be calculated from

$$\phi(t) = y(t) - \eta(t). \quad (10)$$

This definition of the residue recalls that in many curve-fitting procedures, it is some measure of the residue which is minimized instead of, say, a measure of the complementary stimulus, and this suggests the following questions: (D) Have we made the "best" choice of the generator class? (E) How "well" does the response fit the data—or more appropriately and generally—is the relation between the response and the data the "best" one for our purposes?

These are extremely important questions, but we defer complete discussion of them at this time. However, we remark that the answers certainly depend upon the purposes for which the data were gathered and the natures of the physical systems and stimuli involved. The sense of this observation may be made more explicit by the following brief mathematical investigation of the relation between the residue and the complementary stimulus.

III. A BOUND ON THE RESIDUE

Let a class of generators be defined by the functional operator

$$F(\lambda; t) \equiv F_0(\lambda; t) - \sum_{i=1}^{n} a_i F_i(\lambda; t), \quad (11)$$

and suppose it has the property (also possessed by the generator operator in our illustrative example)

$$F(\lambda+\mu; t) = F(\lambda; t) + L(\mu; t),$$

where L is a *linear* functional operator. Suppose that L may be put in the form (as can be done in the example)

$$L(\mu; t) \equiv \mu(t) + \int_0^t K(t, \tau)\mu(\tau)d\tau,$$

where $K(t, \tau)$ is some kernel with all necessary desirable properties. Then, from the definitions of the response, residue, and complementary stimulus, we see that the residue satisfies the Volterra integral equation[9]

$$\phi(t) = \gamma(t) - \int_0^t K(t, \tau)\phi(\tau)d\tau, \quad (12)$$

in which ϕ, γ, and K all depend on the param-

[8] If the w_j can be chosen so that the matrix $\langle w_i F_s \rangle$ is a diagonal one, the calculation of the a_i is greatly facilitated. E. Condon, Univ. Calif. (Berkeley) Pub. 2, No. 4, 55 (1927).

[9] V. Volterra, *Theory of Functionals* (Blackie and Son, Ltd., London, 1931), p. 43.

eters a_i specifying the generators of the class. We suppose that K is such that Eq. (12) may be solved for ϕ in terms of γ, to yield

$$\gamma(t)=\phi(t)-\int_0^t J(t,\tau)\gamma(\tau)d\tau, \qquad (13)$$

where J is the reciprocal kernel. Let us define the "error" E to be the square root of the quadratic norm $\int_0^T\phi^2dt$ of the residue, and we shall suppose that the a_i are determined by minimizing the quadratic norm

$$D^2=\int_0^T\gamma^2dt$$

of the complementary stimulus. Then we ask does the minimization of D also insure that the error E will be a minimum? With the information given we are not able to show this, but we will prove that, (1) if the time of observation T is short enough, E can be made to approximate D as closely as we wish. Moreover, (2) we shall obtain, by the use of Eqs. (12) and (13), upper and lower bounds on the ratio E/D for any T.

From Eq. (12), by squaring and integrating, we have

$$\int_0^T(\phi-\gamma)^2dt=\int_0^T\left(\int_0^t K\phi d\tau\right)^2dt.$$

Then by a well-known inequality, we see that the left-hand side of this equation is greater than, or equal to $(E-D)^2$, whereas by the Schwartz inequality the right-hand side is less than or equal to

$$\int_0^T\left[\int_0^t\phi^2d\tau\int_0^t K^2d\tau\right]dt.$$

Consequently, since $E^2\geq\int_0^t\phi^2d\tau$, we have

$$\left(1-\frac{D}{E}\right)^2\leq\int_0^T\int_0^t K^2d\tau dt.$$

If we denote this upper bound on the right by S^2, we have

$$\frac{1}{1+S}\leq\frac{E}{D}\leq\frac{1}{1-S},$$

for $0\leq S\leq 1$, and

$$\frac{1}{1+S}\leq\frac{E}{D}$$

for $S\geq 1$. Now, since S approaches zero when T does, we see that the statement (1) is proven. By an exactly similar analysis to the above, utilizing Eq. (13) and defining

$$R^2=\int_0^T\int_0^t J^2d\tau dt,$$

we find that

$$1-R\leq E/D\leq 1+R$$

for $0\leq R\leq 1$, and

$$E/D\leq 1+R$$

for $R\geq 1$. Hence, we have the result

$$\frac{1}{1+S}\leq\frac{E}{D}\leq 1+R$$

valid for any period T of observation. This is the result (2). Since R and S both depend on the parameters a_i, we cannot conclude, however, that minimization of the measure of γ, i.e., D, leads to a minimization of the error E; but in the above sense E and D are of the "same order of magnitude."

We conclude with a few remarks on the specific instances in which the method may be of some use. As a matter of fact, it was the consideration of one of these cases which directly motivated the work.

IV. FIELDS OF APPLICATION

The response of a complicated linear electrical network or electromechanical system to a unit emf or unit force applied suddenly to it is often composed of a set of damped and undamped sinusoidal components with various phase relations between them. By choosing a suitable form for the "operational impedance" of the system and by applying the method developed in this paper, one can first find equivalent physical constants of the system and then find an analytical form for the "indicial admittance," i.e., response to a unit stimulus applied suddenly. Then, of course, by the use of Duhamel's integral

the response to any stimulus can be found, or by the use of the operational impedance the stimulus producing a given response can be determined.

One occasionally observes that vibration records of mechanical systems are often subject to error because of drift of the "base-line" of the recording instrument. This drift might take the form of a superimposed vibration or a gross motion representable by a polynomial in the time. It is important to be able to separate out such errors in records and to analyze the records themselves to gain information about the underlying physical phenomena and about the mechanical or electrical recording system involved. In such cases it is possible to apply the method and determine the frequencies and decay constants of the component vibrations, even though they are not rationally related—so that the response record is not a periodic one.

The illumination from periodically variable stars is represented by data which might need to be analyzed to determine the component periods which are present and the relative phases. The application of the method herein described is important in this case if insufficient time has elapsed so that the data have not been collected over a complete fundamental cycle of the phenomenon.[10]

It may in some cases be suspected that certain statistical distributions are describable by some function in which some of the parameters enter in a complicated nonlinear fashion. If the approximating distribution is representable as the solution of some functional equation in which the parameters or an equivalent[11] set enter linearly, then the method may be applied.

A standard problem which arises frequently is one of differentiation of empirical data, and it is often solved by representing the data analytically in some way or other. This is sometimes a rather forced procedure, in that the analytic functions used in the approximation or the parameter values used in such functions do not arise naturally from the physical description of the system that yields the data. It is apparent that by widening the range of possible approximating functions, as is possible using the method herein described, one may hope to avoid such objectionable features. Similar remarks apply to problems of interpolation or extrapolation (prediction) of empirical data.

The record produced by some instrument, responding to a transient stimulus, may be suspected to satisfy, to some approximation at least, a homogeneous differential equation with certain initial conditions. An example might be the response of a good pressure pick-up and recording system to an explosion wave.[12] By transforming the differential system into a single integral relation, the method presented here may be fruitfully applied.

In all these cases definite methods of analysis have been used in the past, and undoubtedly in many instances these will be most practicable in the future. However, one often encounters a situation in which standard methods fail—because in some cases the procedure followed is not a general enough one and thereby "forces" the data, as in trying to fit some data by collocation methods; in other cases, the procedure may require, in principle, the carrying out of an operation such as integration over an infinite range, which is simply not feasible. In the writer's opinion these difficulties might be overcome by the present methods, although one pays for this uniform applicability in that they may not be so convenient or may be more tedious in application to some special cases than some other more specialized techniques would be.

For the comments and helpful suggestions given by M. Rose, A. Douglis, E. Isaacson, G. Whitham, and other associates, I offer my sincere thanks.

[10] Otherwise standard Fourier analysis would undoubtedly be more practicable.

[11] This term is used here to mean that the parameters may be found by the solution of a set of linear algebraic equations or by finding the roots of a polynomial of finite degree whose coefficients are found from linear algebraic equations.

[12] It was exactly this problem, namely the representation of underwater explosion data by a sum of exponentials with unknown amplitudes and decay constants, which first led the writer in 1944–1945 to devise some of the procedures suggested herein.

Richard M. Sutton

Recipient of the 1952 Oersted Medal for Notable Contributions to the Teaching of Physics

The American Association of Physics Teachers has conferred upon Richard M. Sutton, Professor of Physics at Haverford College, the seventeenth of its annual awards for notable contributions to the teaching of physics. The address of recommendation printed below was made by Professor Mark W. Zemansky, Chairman of the Committee on Awards, and the presentation of the medal and certificate was made by Paul E. Klopsteg, President-Elect of the Association, at a ceremony in Sanders Theater, Harvard University, Cambridge, Massachusetts, on January 23, 1953, during the twenty-second annual meeting.

Presentation of Oersted Award to Professor Richard M. Sutton

MARK W. ZEMANSKY,
Chairman of the Committee on Awards for 1952
(Received Februrary 2, 1953)

IT is always one of the most pleasant features of the joint session at the annual meeting of the American Physical Society and the American Association of Physics Teachers to induct into the select fraternity of Oersted medalists one of the outstanding physics teachers of America. The Oersted medal is sometimes conferred upon men at a time near or at the end of their teaching careers. This time, however, our Oersted medalist is relatively young and at the height of his career. Everyone who has attended meetings of our Association, who has given examinations in physics, who has attempted to improve his physics lecture demonstrations, in fact, everyone even remotely connected with any form of physics teaching has been influenced and inspired by RICHARD M. SUTTON.

Descended from a long line of early settlers in America, among whom were several teachers, principals, and college presidents, RICHARD SUTTON spent his boyhood in Denver, his young manhood in Pennsylvania and California, and the main part of his teaching career at his alma mater, Haverford College. At this college he became professor of physics in 1942 and chairman of the department in 1944. His influence as a teacher of physics, however, has extended far beyond the campus of Haverford College. Beside the large number of California Institute of Technology and Haverford undergraduates who were stimulated and inspired to enter the field of physics and who later became first-rate physicists, there is a much greater number of college and high school teachers who owe their inspiration and a good deal of information to the lectures delivered by DR. SUTTON at innumerable meetings of teachers' organizations and at the General Electric Science Fellowship Program at the Case Institute of Technology.

RICHARD SUTTON has served for many years on committees charged with the task of constructing College Board examinations, Graduate Record examinations, and Intermediate Tests for college students. In fact, he might be called "the

grand inquisitor of physics." In addition to this, he has served on many committees of the American Association of Physics Teachers since its founding in 1931. In recognition of his pioneer work with our Association, DR. SUTTON was elected to the presidency in 1940, and is now serving as one of our representatives on the Governing Board of the American Institute of Physics.

It would take too long to enumerate all the boards, committees, and groups of which our Oersted medalist is a member, and to describe all his talents and hobbies. One enormous talent, however, cannot be treated lightly, and that is the clarity, forcefulness, and originality displayed by DR. SUTTON in his demonstration lectures. In this field he is one of the great masters. The laws of physics simply jump out of his apparatus, and however abstract or subtle a physical principle may be, DR. SUTTON has constructed a piece of equipment, or a model, or a figure of speech to make it graphic and understandable. Many of these demonstrations are to be found in a book, compiled and edited by Sutton and published by the McGraw-Hill Book Company, Inc., under the sponsorship of the American Association of Physics Teachers. This book alone has wielded an enormous influence on physics teaching throughout the world and is still serving as the physics demonstrators' bible. It is a commentary on DR. SUTTON's generosity and devotion to physics that the royalties on this book are paid to our Association. Not one penny has ever been accepted by DR. SUTTON.

In addition to his book on lecture demonstrations, DR. SUTTON has recently revised the famous textbook by Mendenhall, Eve, and Keys. With great originality and with a fine feeling for the pictorial aspect of physics diagrams, DR. SUTTON has made this textbook one of the most attractive of its kind.

Although these few facts fail to give a complete picture of the many sides of our Oersted medalist, they are meant to whet your appetite to meet this distinguished man and to hear from his lips some of the influences that were responsible for his reaching the pinnacle of his profession.

I am very pleased, therefore, to ask PROFESSOR SUTTON to receive from the hands of President-elect Klopsteg the Oersted award that he has so brilliantly earned.

The Heritage of a Physics Teacher

RICHARD M. SUTTON
Haverford College, Haverford, Pennsylvania
(Received January 12, 1953)

This article is in response to the presentation of the Oersted Medal of the American Association of Physics Teachers, January 23, 1953. The opportunities and intellectual rewards to physics teachers are stressed.

THIS is the finest birthday present I can remember receiving! To the American Association of Physics Teachers go my sincere thanks for this honor; and to you here present, my thanks for your cordial reception and your willingness to hear a few remarks in response.

Many Oersted medalists in the past have been men retired or about to retire, but there is nothing retiring about the present recipient. He is faced with the prospect of more than a decade of teaching, and his students may rightfully expect him to perform in phenomenal ways. Already, come Monday morning, they will be looking for a new man to appear before them, and they are destined for a disappointment. I expect only the reception customarily accorded to prophets who return to their own countries.

Only once before in my life has January 23rd been such an eventful day, and that was near the turn of the century when I received the gift of life itself. For that event I have had to rely upon hearsay evidence: it was a *cold* night (I am told), and the family doctor who came by bicycle (a bit late for the performance—he was a Harvard graduate) often used to tell me how the wind whistled through the cottonwood trees in front of our home. What chain of circumstances, starting with the arrival of a new human

soul on that night in a western city, could have led to his becoming a physicist and, in particular, a *teacher* of physics?

I propose to examine briefly the heritage of a physics teacher, first by recounting a few personal experiences, and, second, by looking at some of the elements of background common to all of us here, elements which have given us a great inheritance. If this examination should lead to any useful conclusions on how to bring up more and better physicists and teachers of physics, such conclusions will be (I fear) purely accidental.

I.

If a boy were destined to fall in love with Nature, then Denver was a wonderful place to do it. I found myself born into a family that valued the things of the mind and of the spirit, with faith in men and in the power of education, with faith in God and a great interest in His outward garment, this physical world about us. In them was the love of liberty, of freedom of thought, and a strong Quaker sense of responsibility to the best in state and community.

By the dangerous ways in which boys acquire experience and still survive, I learned about an exciting world—about fire, about the flow of water. The boys of the neighborhood followed the horse-drawn engines to every fire within running distance, but any tendency toward arson was held in check by parental restraints. And after every thunder shower, we became hydraulic engineers bent upon stemming the flow of water that rushed down the gutters. I learned about siphons from boys older than myself. Gravity, we discovered not only as a stern master but as a willing ally when, after laboring to push our bicycles to the top of Lookout Mountain, we were repaid by the thrilling six-mile coast down its steep grades. I have the greatest accord with Charley McCarthy's description of such a ride when he says, "About that time, *momentum set in*!" Friction we encountered as the enemy of motion. No boy who has used up his mother's lard supply to grease the ways of a rickety slide from the top of the barn will ever be misled by the neat equations of physics in which friction is neglected.

As a master of the thought-provoking experiment, my father (a former teacher of mathematics) had a good many tricks. I remember, for example, how he would whirl a full bucket of water over his head in a vertical circle, and I found that timidity in repeating this experiment would lead to a bath: *speed* was of the essence, as in many dynamic experiments. What an event it was, one day in the mountains to *see* the speed of sound! Of course, we often heard the delayed clap of thunder that follows lightening, and we understood the cause of the echoes, but this was largely an intellectual matter. But when I saw a distant woodman chopping a tree and heard the reports of his axe when that axe was nowhere near the tree—that was really more like it! And later, to find how much faster than in air sound travels in the steel of a railroad track—truly the world was a wonderful place to explore! This room here in Sanders Theater reminds me of the simple but historic experiments performed with its cushions by Wallace Sabine,[1] experiments that led to the most practical applications in architectural acoustics; and my thoughts hark back to the unsightly maze of overhead wires criss-crossing the Denver auditorium in an early, misguided effort to "break up the sound waves," a system of scientific witch-doctoring used for several years even after Sabine's clear recognition of suitable means for absorbing sound. I attended the Democratic Convention of 1908 in that auditorium, but my taste for politics was not thereby whetted so much as my interest in acoustics and a growing awareness of unsolved problems on all sides.

This world of motion and force and mass had its fascinations. What boy has not called upon the inertial properties of matter to tighten an axe-head on its handle? But *some* of the hazards of motion are, unfortunately, learned only by hard experience. Anything that Isaac Newton found out about gravitation from his legendary observation of the fall of an apple surely made no more impression on him than the dent made on me by the nonlegendary fall I suffered by alighting from a moving trolley car while facing backward. The impact of the back of my head with the street was a quantum of *just* the right size: if it had been any bigger, I should have been completely ionized right on the spot; but

[1] *Collected Papers of W. C. Sabine* (Harvard University Press, Cambridge, 1922).

as it was, I was properly impressed by the law of gravitation and the simultaneous validity of Newton's three frequently stated but not too well-known laws of motion. (I soon thereafter learned to alight from moving street cars with skill, but modern cars with doors have removed this source of education!)

We boys often performed experiments of our own, not always with clear intent, but yet with a steady accumulation of skill and knowledge—knowledge gained by first-hand contact with nature. In those days of crank-winding automobiles, we collected from the garages old dry cells, often by the hundreds. We learned many practical things about electricity long before any of us had heard of Ohm's law, much less of the electron. My mother watched these experiments, perhaps with misgivings, but always with more insight than I credited to a mere woman. Her father had been a professor of Natural Philosophy at Haverford before there were professors of physics, and she doubtless acquired knowledge of things scientific by osmosis rather than by formal training. She had a way of asking leading questions but of providing no answers. On one occasion when, never having heard of Michael Faraday, I was trying to pass a current through a mixture of salt water, vinegar, and anything else I could find, for no good reason except to see what might happen, I encountered the acrid odor of chlorine. But, of course, electricity could have no *possible* connection with chemistry, and I scoffed at my mother's suggestion that there was any!

It was by steps like these, wherein much was left to the natural curiosity of a boy, and encouragement was given to independence of thought, that I found myself at the age of sixteen entering into a high school course called Physics. My awakening was not unlike that of Molière's *Bourgeois Gentilhomme* who discovered to his amazement and delight that he had been talking in prose all his life. So this was called physics?—this, my natural food and drink which had previously gone unnamed. Millikan and Gale's *First Course in Physics* was (I felt) written just for me; and under the clear-minded guidance of E. Waite Elder, a man of extraordinary skill as a teacher, the flame of casual interest was fanned into an active blaze. For two years thereafter it was my good fortune to serve as Elder's laboratory assistant, with further opportunity to experiment, observe, and test. Our association was crowned by a journey to the top of the Continental Divide to make magnetic observations during the solar eclipse of June, 1918.[2] Appropriately enough, we went to a snow-shed town called Corona which now, since the opening of the Moffat Tunnel, is only a charred memory. The leader of our three-man expedition was Dr. Louis A. Bauer, then director of the Department of Terrestrial Magnetism. His tales of the good ship "Carnegie," and of his world-wide travels in the study of magnetism (years before the hey-day of cosmic-ray globe trotting!) made a big impression on a young man about to enter college.

After a year at the University of Colorado, I turned toward Haverford College with the avowed intention of majoring in physics. My fellow students looked on me with some misgivings, for Frederic Palmer, the professor of physics, was at that time the feared and revered Dean of the College; and one did not approach his precincts without full armor and a large supply of vanishing cream. But three years of work with him as a student, followed (after an intervening decade) by thirteen years as his colleague, have shown me the error of the prevailing student view of the 1920's and have impressed upon me the sterling worth of the man as scholar, teacher, and friend. No man ever extracted so much juice from an experiment: his lectures in atomic physics were masterful and opened to me a region previously glimpsed only imperfectly from afar. How often he ended his lectures with a *question!* For him and for his students there was something more that deserved investigation.

During my first year or two of teaching at Miami University, I was often mistaken for a Freshman, until my face began to work its way up through the top of my head. I found in my first boss, Professor J. A. Culler, a gentleman of the old school whose physics, though dating from the 1890's, was sound and thorough. He used to say, "What happens in an experiment *should* happen." How often his point of view

[2] The second return of this eclipse in its 18 year and 11 day period of the Saros will begin on the morning of June 30, 1954. The path of totality will start in Nebraska and pass over Minnesota, Wisconsin, northern Michigan, and Canada.

has helped me to turn disaster into triumph by capitalizing on unexpected turns in demonstration experiments, rather than by being defeated or embarrassed by them.

When the right time came for graduate work, I sought out R. A. Millikan. The things that one learned from "The Chief," as he is known at Cal Tech, were numerous, not the least of which were those connected with the courses he taught. When one has worked with a man for several years, one is likely to be fondly partisan, despite knowledge of the varying degrees of regard in which he is held beyond his own domain. From Millikan, I brought away a point of view that might be paraphrased: "Ask an intelligent question of Nature, then seek an intelligible answer." Seeking for a detailed experimental answer is, of course, at the heart of progress in research; but such seeking must be guided by persistent and intelligent questioning.

One final word of personal history should suffice. From what I have said, it might be concluded that an active interest in the physical world may spring up quite early—perhaps it is innate, and perhaps there is nothing that can prevent it from developing. But what of a primary interest in *teaching*? If one were to become a teacher by inheritance, then I had every opportunity; but the resolve to follow the profession of teaching was, I feel, not just inherited. It came clearly and definitely soon after the end of my first year at Haverford—I could almost place a date on it. It came as a conscious realization that one could, in a teaching career, combine an absorbing interest in nature with a deep interest in people, and that one could thereby stake out a claim in the great territory of life—a claim not limited by any discernible horizon, for the territory of the mind and spirit is vast and freer from fences than the Rocky Mountains we love so well.

II.

About our general and common heritage I shall be brief.

From the universe of interest that is presented to each new citizen of the world, those here present have sought principally after an understanding of the nature of the physical world. How many-sided is this world of ours! To the inquiring mind it presents endless frontiers. To the research worker it slowly yields its secrets but gives the impression of withholding more than it reveals. Each of us is often convinced of his own profound ignorance; but we need not be dismayed, for the slow whittling down of ignorance is the best evidence of the upward march of mankind. We follow a profession which year by year shows significant advances. To some is granted the satisfaction of contributing to these advances. To all is presented the challenge of seeing that these advances shall be used constructively, for the good of all men.

But to the teacher of physics, this great inheritance seems to be poured out in extra measure. Ours is the opportunity to associate with the Galileos, the Newtons, the Faradays, the Maxwells, the Rutherfords, and our contemporaries, to make it our business to understand their work, and to bring each new generation of students into an appreciation of their accomplishments. Ours is the task of putting *life* into the fabric of science and of seeking ways to bring students quickly to the level of creative and independent work, this, despite the almost boundless and accelerated growth of our science. Ours is the privilege of associating year after year with the young people of our population, that steady stream of raw material from which must come the men and women who will maintain and extend the technical structure of our civilization.

Ours is the responsibility of working at that vital point where the human element in the development of science and of scientists is uppermost. We find ourselves the recipients of a proud past, the interpreters of a remarkable but often baffling present, and the sculptors of a great future, wherein the progress of the race may well depend more upon how we solve some of our pressing problems as *people* than upon the making of faster means for solving complex equations or the contrivance of more intricate electronic circuits or the construction of more powerful particle accelerators. Truly, the legacy of a physics teacher is a great possession, to be cherished, to be actively developed, and to be transmitted to a new generation that, happily, shall long outlive the transmitter.

Electronics in a Sophomore Physics Course

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(Received June 9, 1952)

The introduction of electronics in a sophomore physics course is discussed with special emphasis being given to the content of the half-year course in electronics, electricity, and magnetism. Experiments and equipment are listed and several are presented in some detail.

THE extensive development of electronic instruments which has revolutionized procedures in industry has influenced methods of measurement in the physics laboratory as well. Therefore, it has seemed appropriate to introduce their use in the physics program at the sophomore level. This might be difficult in a conventional physics program for engineers and physics majors; however, with increased emphasis on "core and general education" requirements and a concurrent minimizing of preprofessional offerings, the opportunity presented itself in the Santa Barbara College of the University of California. Majors in physical science and physics now take an 8-unit course in general college physics in the freshman year. This course is not designed as a special course for such majors, however, but serves students of the biological sciences and others who may elect it.

To assure adequate preparation for advanced undergraduate work, an intermediate two-semester course of eight units, which includes two units of laboratory, has been introduced for the physics major. One semester is required of physical science majors. Electronics is introduced at the beginning of the semester and is followed by conventional study in electricity and magnetism. The second semester of the course includes selected topics in acoustics, optics, and mechanics. This article will be a consideration of the first semester of four units with special emphasis on the electronics portion.

Approximately one-third of the lectures are devoted to the study of electronics. Though this time is short, it permits sufficient discussion of theory and instruments to provide background for the experiments. Selected topics of the textbook by Hill[1] are assigned. The order of presentation is as follows: D'Arsonval meters, student-type potentiometer and calibration, ac circuits, electron emission, triodes, load line and amplification, tetrode, pentode, beam power tubes, gas-filled tubes, oscilloscope, amplifier, vacuum-tube voltmeter, rectifiers, and filters. New books by Geppert[2] and Mitchell[3] have served well for supplementary references, and the latter might be taken as a text, assuming careful selection of the material presented. The remainder of the first course is devoted to a study of electrostatics, magnetostatics, including dielectric and magnetic media, current electricity, and electromagnetic waves in free space. Frank's[4] textbook has been used for this portion.

Considerable emphasis has been placed upon the laboratory work; it is here that the greatest divergence from the conventional course occurs. Since no suitable manual has been found, mimeographed guides to experiments have been prepared. Although the accent is upon the teaching of fundamentals, concepts, and characteristics of components, the acquisition of competence in the use and application of electronic instruments and equipment is also an important aim. To this end the full three-hour period of the laboratory is utilized for experimentation, and the time required for the written report is kept minimal. Circuit panels and convenient connecting arrangements contribute toward an efficient use of the student's time, yet require him to make necessary circuit connections. He learns to use new tools in the attainment of new knowledge. An attempt is made to prepare the student for the work of the modern physics laboratory

[1] W. R. Hill, *Electronics in Engineering* (McGraw-Hill Book Company, Inc., New York, 1949).

[2] D. V. Geppert, *Basic Electron Tubes* (McGraw-Hill Company, Inc., New York 1951).

[3] F. H. Mitchell, *Fundamentals of Electronics* (Addison-Wesley Press, Cambridge, 1951).

[4] N. H. Frank, *Introduction to Electricity and Optics* (McGraw-Hill Book Company, New York, 1950).

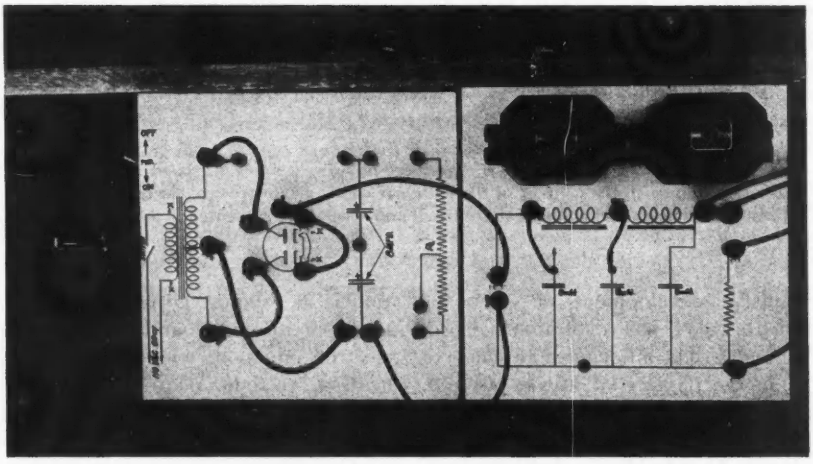

FIG. 1 Rectification and filter panels showing GR terminals and circuit. Panels are connected for filter experiment with full-wave rectificaton and capacitor input filter. Half-wave, full-wave, and voltage-doubler rectifier, one-and two-section condenser or choke input filter connections are available.

of the junior year wherein a measure of independence and self-reliance is expected.

The titles of the experiments follow: (1) Calibration of Milliammeters; (1a) Modification of the Range of a Milliammeter; (2) Characteristics of a Triode Electronic Tube; (3) Characteristics of a Pentode Electronic Tube; (4) Use of a Cathode-Ray Oscilloscope and an Audio Oscillator; (5) A study of Rectification Circuits; (6) Power Supply Filters; (7) Study of a Resistance-Capacitance-Coupled Voltage Amplifier; (8) Characteristics of Gas Tubes; (9) Study of Alternating Current Circuits; (10) Calibration of a Ballistic Galvanometer; (11) Hysteresis Measurements; (12) Bridge Method of Capacitance Measurement. During one semester an inspection of an electronic computer and a visit to a display and demonstration of commercial electronic instruments were combined in a field trip.

An inventory of the instruments and panels used is given below, exclusive of such items as rheostats, resistance boxes, and components. Numbers following the item indicate the experiments in which it is required. After the first five experiments, some rotation is necessary due to limitations of equipment, but the listing follows the general order of introduction of instruments.

Equipment: Leeds and Northrup student-type potentiometer, 1; galvanometer—portable, 1; standard cell, 1, 10; milliammeter—dc, 1, 1a, 2, 3, 5, 6, 7, 8; Leeds and Northrup type S test set, 1a; electronic circuit panel—modified Kepco type, 2, 3, 7, 8; voltmeter—dc, 2, 3, 6, 8; triode, 2, 7; power supply, 2, 3, 7, 8; pentode, 3; vacuum-tube voltmeter, 3, 4, 5, 6, 7, 8, 9; cathode-ray oscilloscope, 4, 5, 6, 7, 8, 12; audio oscillator, 4, 5, 7, 12; voltmeter—ac, 5, 9; multimeter (Simpson), 5; rectifier panel (and duo-diode), 5, 6; selenium rectifier panel, 5; load panel, 5, 6; milliammeter—ac, 5, 8; filter panel, 6; square-wave generator, 7; neon lamp, 8; voltage regulator tube, 8; thyratron tube, 8; variac, 8; resonance panel—Crow capacitor and resonance coil (60 cps), 9; ammeter—ac, 9; wattmeter, 9; galvanometer—ballistic, 10, 11; ammeter—dc, 10, 11; condenser—standard, 10, 12; solenoid—standard, 10, 11; search coil, 11; ring sample solenoid and coil, 11; impedance bridge (Brown), 12; capacitance bridge General Electric YCW-1, 12.

The titles and list of equipment suggest the content of the experiments. Experiment 1a is a continuation of the milliammeter study of the first laboratory period. After determining the sensitivity of a war-surplus milliammeter, a shunt is constructed and the meter mounted in a case. Silver soldering, use of chassis punch, and calibrating techniques are introduced. The studies of triode and pentode tubes are standard electronic experiments with the vacuum-tube

voltmeter, having been previously discussed in the lecture, introduced in the second of these.

Special panels pictured in Fig. 1 have been made for the rectification and the filter experiments. The rectifier panel utilizes a war-surplus transformer with a 530-volt secondary, center-tapped, and a 117Z6-GT rectifier tube. Both half-wave and full-wave connections are made. (The panel also provides a voltage-doubler circuit used in an upper division course). The condensers are used in the rectifier experiment to show smoothing of the ac voltage component. This component is measured by means of a calibrated oscilloscope and a vacuum-tube voltmeter. The filter experiment, performed after the rectification circuits have been studied, requires both panels. A full-wave rectification circuit with a capacitor input filter is shown in the figure. The chokes are United Transformer Company type S-30 swinging and S-29 filter, 90 ohms, 175ma. Selected spot comparisons of single section choke and capacitance, as well as resistance-capacitance filters, are made. In addition, data are taken for voltage *vs* load curves for both standard types of two section filters. Convenient circuit conections which are readily changed improve the laboratory efficiency.

Experiment 7 combines a square wave generator in addition to the sine wave oscillator and vacuum-tube voltmeter in the study of the voltage amplifier. The square wave generator is first used to investigate the frequency response of the oscilloscope. The amplifier circuit is then connected as shown in Fig. 2. Plate voltage and current are determined and a self-bias of 4–7 volts established. An audio oscillator supplies a sine wave and the gain is determined for a limited number of frequency values. Finally, the square wave generator is connected between the oscillator and the amplifier and investigation of the frequency response made, utilizing the capabilities of the oscilloscope. (Study with the square wave generator might well precede the gain measurements, but the operation of the amplifier should definitely be tested with a sine wave first). Overdriving is shown in a class demonstration but left optional in the laboratory.

Comparison of neon-lamp firing potentials with ac and dc voltages, voltage regulator and thyratron characteristics, and sweep-voltage production are incorporated in one experiment. An OA3/VR75 regulator tube with protecting resistance and jumper is connected in the circuit panel by the student and a voltage *vs* current relation established. After determining the characteristics of an 884 thyratron, a condenser is connected across the plate-cathode circuit and with a selected bias setting the plate voltage is adjusted for a good sweep wave form as shown by the oscilloscope. Time does not permit synchronization but this feature is discussed.

The last four experiments approximate standard physics experiments except for the incorporation, where possible, of commercial measuring instruments. Bridge circuits for capacitance measurements are set up with standard equipment before the commercial instruments employing the circuits are used.

The Kepco[5] type electronic circuit panel which is shown with a typical amplifier circuit connection in Fig. 2 has been constructed using GR

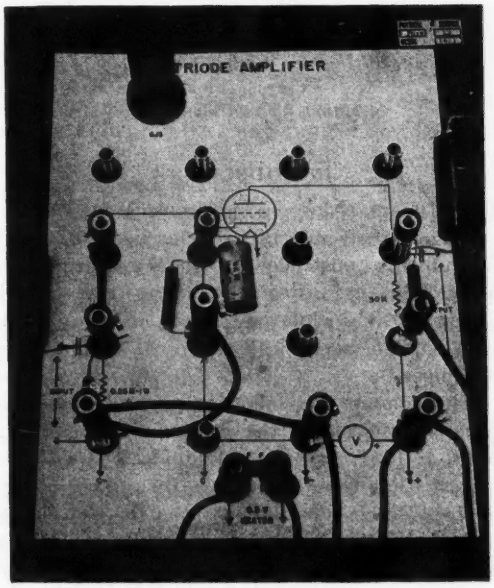

FIG. 2. Universal circuit panel of modified Kepco type, shown here with a voltage amplifier circuit connected for experiment 7. GR terminals and plugs permit rapid conversion of circuits. R_L 30 000 ohms; R_k 1000 ohms; input and coupling capacitors 0.1 microfarad.

[5] Kepco Laboratories, Flushing, New York.

terminals. These panels have been used for several years in other courses and have proved very satisfactory. The GR-type binding posts and plugs withstand usage well and permit rapid, positive connections to be made. These connections are standard in the laboratory. Each group of students maintains its own set of wires and stores them in a tray. Oscilloscopes and vacuum-tube voltmeters of various makes, both complete and purchased in kit form, are in use. Heathkit and Eico instruments appear to offer a reasonably satisfactory means of providing a number of meters at low cost.

The program outlined permits the student to learn, early in his career, the operation and, to some extent, the theory of operation of a number of useful "tools of measurement." Through frequent reference to instruction manuals he is assisted toward self-instruction. Circuit reading is introduced indirectly. Essential knowledge is immediately applied both in theoretical consideration of circuit applications and in the laboratory experiments. It is believed that the student will profit from early acquaintance with electronic instruments and will be more adequately prepared for specialized applications in advanced undergraduate courses and for experimentation dictated by his own interests.

The Physics of the Organ Flue Pipe

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(Received July 28, 1952)

This is a review article in which the physical principles underlying the function of the organ flue pipe are considered. Many of the points are not understood, and attention is called to them as possible lines of investigation. A survey is made of the various theories of the action of the airstream at the mouth of the pipe; details are given of the variations in construction and their effect on the tone. Finally attention is paid to the tones of organ pipes in combination, and the conditions for a satisfactory musical effect are outlined.

INTRODUCTION

THE organ-flue pipe is a musical instrument of great antiquity. References to small instruments constructed on the same principles as modern organs are found in the second century B. C., while pipes from an organ built *ca* A. D. 226, are basically identical with modern flue pipes.[1]

Throughout the following centuries steady progress was made; the organ at Halberstadt, built in 1361, had several hundred pipes, the largest being some 25 ft long. By the beginning of the 17th century the organ on the Continent had practically reached the modern form; descriptions are extant[2] of instruments which would even today be considered large organs. Tonal development reached a peak of excellence on the continent of Europe in the 17th century (referred to later as the "classical period"); speaking very broadly, these instruments were characterized by many ranks of gently speaking pipes, giving an impression of brilliance rather than of great power. Development was relatively slight until the middle of the 19th century, when rapid advances were made; these were initiated and chiefly carried out by the organ-builders Aristide Cavaillé-Coll in France, and "Father" Henry Willis in England. Higher and steadier wind pressures were used; pipes were constructed in different ways giving new tone-qualities, and ideas of ensemble were in many cases transformed. Since about 1890 advances in tonal development have been relatively slight, but the application of electricity to organ mechanisms has revolutionised the mechanical aspect of the instrument.

In the past there are occasional instances of the collaboration between organ-builder and

[1] W. L. Sumner, *The Organ, its Evolution, Principles of Construction, and Use* (Macdonald and Company Ltd., London, 1952).
[2] Michael Praetorius, *Syntagma Musicum* (Wolffenbüttel, 1619; facsimile edition, Kassel, Germany, 1929).

scientist. It is not generally known that the organ-builder Arp Schnitger assisted von Guericke, the Burgomaster of Magdeburg, with his vacuum pump; and, much later, Foucault's apparatus for determining the velocity of light was constructed by Cavaillé-Coll. But the making and treatment of the pipes has remained entirely a craft, empirical knowledge handed down from generation to generation, and the uniform excellence of the product seems to indicate that the scientist's assistance is unnecessary. The organ pipe has received little scientific attention and, as will be seen later, even the elements of its functioning are not at present understood.

It is due to such facts that the author feels some explanation of this paper to be necessary. A review paper frequently presents a coherent picture; fundamental facts are readily explained, and an outline of much of the subject is fairly complete, with indications of the directions in which development is needed. The present paper falls lamentably short of such a standard. Many of the points are imperfectly understood, and several of the known facts are entirely without explanation; the situation, in fact, has not greatly changed since Maxwell wrote of "that untrodden wild between acoustics and music, that Serbonian bog where whole armies of scientific musicians and musical men of science have sunk without filling it up."[3] So much so, in fact, that it seemed desirable to draw attention to such points which are worthy of investigation, and this may be taken as one of the aims of this paper.

One further point may be mentioned. In discussing such a vast subject, only normal practice can be dealt with. There will be exceptions to all of the statements made about ordinary usage; this note will, it is hoped, save constant repetition of such qualifying statements.

THE ORGAN: TONAL STRUCTURE

The organ is a keyboard instrument in which a number of tone-qualities may be controlled from one keyboard. These are the "stops." Each stop of the organ consists of a rank of pipes, one pipe to each note of the keyboard, all of approximately the same timbre and power. By "drawing" the stop (moving a knob or tablet near the keyboard) the effect is that when the keys are depressed, the pipes of only that particular stop are sounding. This stop may be withdrawn and another substituted, or any number may be used together. An organ is normally provided with more than one keyboard, or "manual," together with a keyboard played by the feet, for the bass pipes; the number of stops controlled by each keyboard varies from a few to a score or so in a large organ, but the principle remains that when a key is depressed, only the pipes of the stops which are drawn are made to sound. We shall see directly how this is carried out.

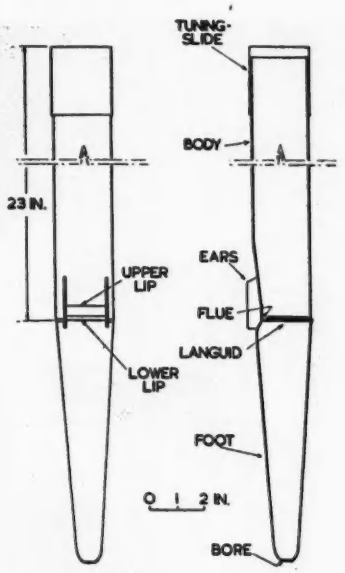

FIG. 1. Scale drawing of open diapason pipe, middle *C*.

A rank of open pipes has the pipe corresponding to the lowest note, two octaves below middle *C*, approximately 8 feet long. Such a stop is called an 8-ft stop. It is essential, for musical reasons, to introduce stops corresponding to other pitches; these are still designated by the length of the lowest pipe, so that a stop speaking the octave pitch would be a 4-ft stop, and so on. This point will be returned to later.

WINDCHESTS AND ACTION

The part of the organ which distributes wind to the pipes can have a significant effect on the

[3] J. Clerk Maxwell, *Scientific Papers* (Cambridge University Press, Cambridge, 1890), Vol. II, p. 754.

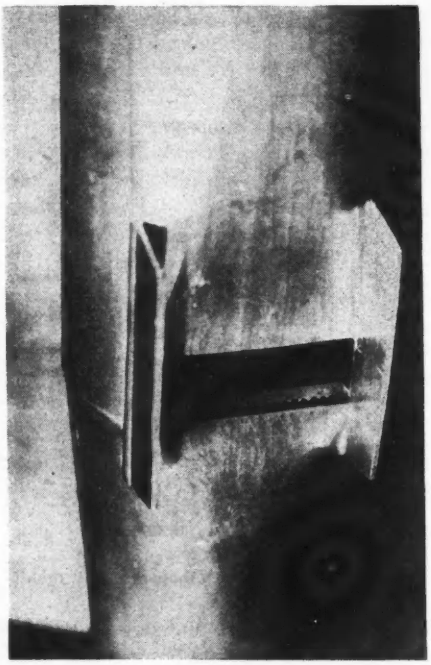

FIG. 2. Photograph of mouth of open diapason pipe, middle *C*.

tone of the instrument. Details of mechanism may be found in the descriptive textbooks.[4,5] The salient difference is as follows; in the traditional slider chest a large valve opens into a chamber from which a number of pipes are fed, while in most modern mechanisms each pipe has its own valve, opening into a small chamber immediately below the pipe-foot. The rate at which the pressure rises in the pipe-feet will be quite different in the two cases; it is shown later that variations in the initial speech have a powerful effect on the tone of a pipe, and it is known that pipes generally need adjustment if transferred from one chest to another.

It would be valuable to have a thorough investigation of the build-up of pressure in the pipe-foot.

[4] N. A. Bonavia-Hunt, *The Modern British Organ* (A. Weekes and Company, London, 1950).

[5] W. H. Barnes, *The Contemporary American Organ* (J. Fischer and Brother, New York, 1948).

DESCRIPTION OF ORGAN PIPES

Organ pipes are constructed in many different ways, but the open metal cylindrical pipe may be considered the fundamental type; it is known as the "open diapason," and produces the tone which characterises the church organ. Such a pipe, in the middle *C* region, is shown in Fig. 1. The languid seals the junction between the foot and body, except for a slit, known as the flue, between the straight edge of the languid and the "flatted" upper part of the foot. A rectangular aperture is cut in the body, and two "ears" frequently fitted. The generic name for all this part is the "mouth"; a photograph of the mouth of a diapason pipe is shown in Fig. 2.

The mode of action will be detailed later, but may be briefly described here; "wind," i.e., air under pressure, flows through the bore into the foot, and out through the flue. There is an interaction between the issuing lamina of air and the edge of the upper lip, exciting the natural frequencies of the air in the pipe-body, which behaves as an air-column with both ends open.

The steady tone of an organ pipe consists of a fundamental and a series of harmonics. It is

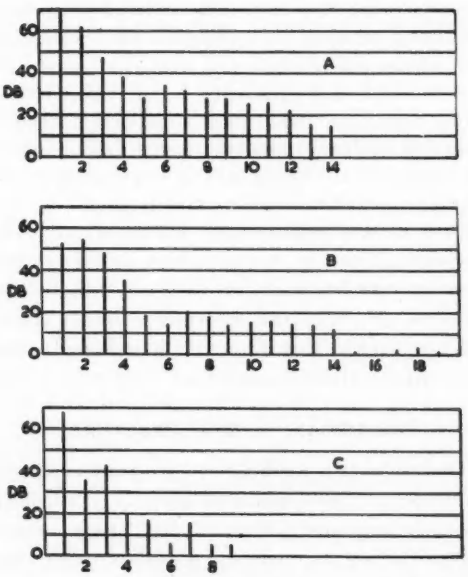

FIG. 3. Typical spectra of open flue pipes (middle *C*); intensity plotted against harmonic number. (A) open diapason; (B) string; (C) open flute. Zero level of intensity is arbitrary, but the same in each case.

convenient to use the term "harmonic development" to indicate the number and intensity of these harmonics, and to consider diapason tone as that of moderate harmonic development, thus providing a convenient norm.

A pipe of high harmonic development will have a "brighter" or "keener" tone, such as is given by so-called "string" pipes. Conversely, a pipe of low harmonic development, a so-called "flute," has a "duller" tone than a diapason. Epithets such as "stringy," "fluty," etc., are common. This is of course, only a broad classification. Diagrams showing the harmonics of typical pipes are shown in Fig. 3.

The tone of a pipe is, however, by no means specified only by the steady-state spectrum; the manner in which the tone builds up at the start or the "articulation," is of the highest importance. The pipe may emit noise, of a more or less definite pitch; a musical note, whether one of the harmonics or not, may be prominent for a short time. Also the pipe may be "slow," i.e., build up the note gradually, or "fast," suddenly attacking its note. It should be noted that such anticipatory sounds are seldom heard as such, although they have a marked effect on the apparent tone of the pipe; also they are not necessarily defects. Any type of transient may be desirable under certain voicing conditions. Typical transients are shown in Fig. 4.

Two further factors affecting the steady tone may be introduced here. Random noise in the tone of a pipe may have a considerable effect, varying with its frequency distribution. A more important one is the effect on the tone due to vibrations of the pipe-body. Little is understood about the physical side of this, although the organ-builder can combine the various factors of wall material, thickness, etc., and impart to the tone of a stop of pipes a highly desirable quality, generally known as the "formant."

The process of "voicing", to which reference is made later, represents the final process in making an organ pipe give its proper tone; it consists of small adjustments, made chiefly to the mouth parts and the bore.

PHYSICAL INVESTIGATIONS

It is surprising that the organ pipe, which had been so familiar for hundreds of years, attracted

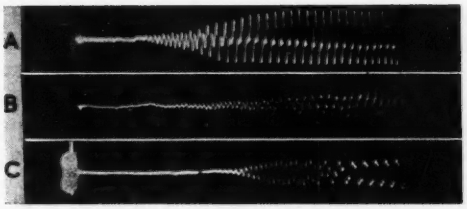

FIG. 4. Typical transients of flue pipes (middle *C*); (A) open diapason, normal, (B) open diapason, abnormally slow; (C) lieblich gedackt.

no attention from scientists until 1826.[6] In that year the brothers Weber[7] advanced the following explanation: the first puff of wind from the flue starts a compression which travels up the pipe, is reflected at the end, travels down, and reacts on the blowing wind. Compressions and rarefactions are set up in the wind supply arriving at the flue, and these maintain the vibration. While this explanation is evidently on the correct lines, it seems, at present, that the reaction on the wind supply is a secondary effect.

It was apparently in 1840 that Cavaillé-Coll[8] introduced the concept of a "free aeriel reed" (anche libre aérienne). This was the first mention of the celebrated "air-reed" theory, which held the field for so long. The fullest development of this theory is in the writings of Hermann Smith[9,10] (together with a mass of irrelevant and sometimes humorous material). It was evidently widely accepted in the 1870's; the explanation of Helmholtz is probably the clearest. He says[11] "The blade-shaped sheet of air at the mouth of the organ pipe is wafted to one side or the other by every stream of air which touches its surface. . . . When the oscillation of the mass of air in the pipe causes the air to enter through the ends of the pipe, the blade-shaped stream of air arising from the mouth is also inclined inwards, and drives its whole mass of air into the pipe. During the opposite phase of vibration . . .

[6] A. T. Jones, *Sound* (D. Van Nostrand Company, Inc., New York, 1937). This book contains an extensive account of early investigations of the organ pipe.

[7] *Wilhelm Weber's Werke* (Julius Springer, Berlin, 1892), Vol. I, p. 166.

[8] A. Cavaillé-Coll, Compt. rend. **50**, 176 (1860).

[9] H. Smith, Nature **8**, 25 (1873); **10**, 161, 481 (1874).

[10] H. Smith, *The Making of Sound in the Organ and in the Orchestra* (Reeves, London, 1911).

[11] H. L. F. Helmholtz, *On the Sensations of Tone* (Longmans, Green, and Company Ltd., London, 1895), third English edition, p. 92.

when the air leaves the pipe the whole mass of this blade of air is driven outwards. Hence, it happens that exactly at the times when the air in the pipe is most condensed, more air still is driven in from the bellows, when the condensasation . . . is increased, while at the periods of rarefaction in the pipe the wind of the bellows pours its mass of air into the open space in front of the pipe."

This theory certainly accounts for the maintenance of the tone; it may be remarked that it does not explain the initiation. Helmholz went on to point out that since the "air-reed" takes time to vibrate, it delivers energy to the pipe in pulses (to use modern terminology), and he deduced that such a form of vibration would excite the higher harmonics of the pipe.

Further experiments[12] on this point, made by introducing talcum powder into the blowing wind, were not very conclusive, but van Schaik,[13] in a prize essay followed up the work, and showed that the windstream did in fact vibrate as a reed, in and out of the pipe.

Other work done under Wachsmuth at the beginning of the century does not appear to have carried matters much further. An important advance was made when edge-tones alone, without a resonator, were studied. In these, air from a slit was blown onto an edge, and it was found that tones could be produced; the general relation was that the frequency was inversely proportional to the slit/edge distance, but different "series" of tones were also found.

Wachsmuth also introduced ether vapor into the windstream, and was able to take spark photographs of the motion. By this means he was able to show that vortices were formed, and that the frequency of the note was the same as the rate of formation of the vortices. This represented an important advance, which was given still greater impetus when Kármán published his theories of the vortex street known by his name. The edge-tone theory which was developed,[14] and is still current, is that the vortices in the blowing wind form one side of the vortex "street," the other side being formed by secondary vortices which are generated at the edge, and which fall into step with the others. The theory postulates that the rate of striking the upper lip of the pipe by these vortices constitutes the edge-tone: this tone, which is near to the fundamental of the pipe, causes it to resonate. It is possible in this way to explain several points in the behavior of the pipe at different wind pressures.

Several experiments have been made more recently on edge-tones,[15,16] but there is disagreement not only on the postulated mechanism, but also on the results, and the situation remains inconclusive. (Minute differences in the apparatus, such as the streamlining of the air-passages, might account for some discrepancies.) It appears to the author that great caution must be exercised in applying edge-tone theory to the organ pipe. In the edge-tone experiment, the jet blows symmetrically against the edge, which is in free space: but in the organ-pipe the jet is not only directed somewhat outside the pipe, but is also bounded on one side by the strongly resonant pipe-body. Further, the edge-tone frequency which would be given by a similar jet and an edge set in the same position as the parts of the pipe is often higher, by more than an octave, than the fundamental of the pipe.

The situation at present is extremely difficult to understand. The aerodynamic problem of a plane jet which is disturbed is a very complex one. It seems that only an approximation to a Kármán street can be formed, since in theory its extent must be infinite, and at the mouth of an organ pipe there may be only three or four vortices. The author's inclination is to consider that the early "air-reed" theories have a good deal of truth in them, but the vortices play an important part in directing the reed; possibly the reed is, in fact, formed by the vortices.

THE CONSTRUCTION OF ORGAN PIPES[17]

Material

The usual alloys contain tin and lead only; "plain" metal with some 5–20 percent tin,

[12] V. van Tricht, J. de Physique **6**, 53 (1877).
[13] W. C. L. van Schaik, Archives Neederlandaises **25**, 281 (1892).
[14] E. G. Richardson, *Sound* (Edward Arnold & Company, London, 1947), Chap. 6, 7.
[15] G. B. Brown, Proc. Phys. Soc. (London) **49**, 508 (1937). This paper contains a full bibliography.
[16] E. G. Richardson and J. M. Lenihan, Phil. Mag. **29**, 400 (1940).
[17] D. M. A. Mercer, J. Acoust. Soc. Am. **23**, 45 (1951). This paper contains a more detailed description of the construction and voicing of organ flue pipes.

"spotted" metal (so-called because the surface is covered in "quasi-crystals") with 50-60 percent tin, while "pure tin" contains in fact only about 92 percent of tin. Some 1–2 percent of antimony is sometimes added to harden a low tin alloy. Zinc is widely used, mainly for pipes longer than 4 ft; copper is occasionally employed.

The sheet of metal from which the pipes are made is obtained by casting on a stone slab covered with fustian, the metal being poured at such a temperature that nuclei of one phase are present. The metal is then scraped and planed to thickness by hand, and the pipe formed to shape over mandrels. The relatively light pressures during these processes do not affect the metal structure adversely, but methods of machining the metal to thickness, or rolling it out, evidently have some effect which deadens the tone. The metallurgical problems of the whole process, from the casting—a method which is several hundred years old—to the final metal structure and mechanical properties, should make a particularly interesting investigation.

The whole question of the effect of wall material on tone is a difficult one, and investigations which have been made are not very conclusive.[18,19] The practical organ-builder uses the higher tin alloys for bright-toned pipes, and low tin alloys for broader tones, as a regular thing; but he is not limited to using such materials, and satisfactory tones can frequently be obtained from almost any of the normal materials. This is another point which would bear a thorough examination; the main obstacle being the extreme difficulty of constructing two pipes of different materials but exactly alike in all other respects.

Wall Thickness

Wall thickness is mainly determined by the power required from a pipe; attempts to produce a powerful tone from a thin-walled pipe result in a harsh, unmusical tone, and it is significant that this can often be cured by soldering a collar of metal round the center of the pipe body. On the other hand, a quiet-toned pipe needs thin walls, as a thicker wall could not be set in vibration, and the tone would consequently suffer.

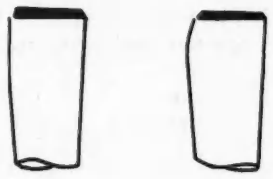

FIG. 5. Languid and lower lip formations: (A) Straight-flatted lip, blunt languid; (B) Sharply-curved lip, sharp languid.

The relationship between wall thickness and material is a very important one, and one clearly understood empirically by builders. It probably has some bearing on the relation between material and tone, since, e.g., it is clearly not practicable to make a thin-walled pipe of soft metal.

Diameter

The diameter of a flue pipe is one of its most important properties. In organ building parlance it is usually termed "scale." Diapason pipes have moderate scales; the diameter of the middle *C* pipe (2 ft long) principal open diapason for a large instrument would be about $2\frac{1}{4}$ in. Flute pipes of the same frequency would probably be larger, and string pipes perhaps as small as 1 in. in diameter. Power and harmonic development are settled largely by the scale of the pipe; a large scale gives a more powerful note, but a duller tone. (The precise relationship will be considered later.)

Mouth; Dimension

The exact effect of mouth-width is not very clear, but it is general practice to give wide mouths to bold-toned pipes, and narrower mouths to softer pipes. A diapason for a resonant building might have a "2/7 mouth"; this indicates that the width of the mouth is 2/7 of the circumference of the pipe. More usual widths are 1/4 and 2/9, while 1/5 is used for quiet stops. There are however many exceptions.

Generally the wider the mouth, the lower the "cut-up" i.e., the height of the mouth, needs to be for good tone. A 1/4 mouth might have a

[18] W. Lottermoser, Akust. Z. 2, 129 (1937).
[19] C. P. Boner and R. A. Newman, J. Acoust. Soc. Am. 12, 83 (1940).

cut-up of 1/3 of its width; to increase the cut-up of a diapason pipe too much is risking a dull and "lifeless" tone.

Nicking

The practice of nicking the languid and lower lip is apparently some 300 years old. It has a very powerful effect on the tone; an unnicked pipe may "whistle" or "scrape," producing a rough sound and being slow to produce its steady note, while the note will be rough and harsh. Correct nicking will produce a pipe which initiates and sustains its note in a smooth and musical manner, but excessive nicking gives a tone which, although smooth, is somewhat dull. There is some evidence that the random noise in the tone of a pipe is reduced by the nicking.

Languid and Lower Lip

Up to about 200 years ago, a blunt languid with a "straight-flatted" lower lip were used. These, combined with a thin-walled pipe of low tin alloy, low wind pressure, and light nicking, produced the characteristic, somewhat fluty tone of the early English organs; it is occasionally reproduced today. For the higher wind pressures and bolder tones required now, a sharper-angled languid and more curvature to the lower lip (sometimes accented by "dubbing" it in to make it curve sharply near the languid) are used. Different types are shown in Fig. 5.

The reasons for this behavior would bear investigation. The blunter languid directs the windstream more out of the pipe; the effect on the airflow of the two constructions might well be examined in detail.

Bore Diameter

This has a direct effect on the quantity of wind supplied to the pipe (in conjunction with the width of the flue). It must, however, be considered in relation to other variables. For instance, a pipe may be placed on a higher pressure, and the bore closed up so that the speaking pressure is the same as before; but the rate of filling up of the foot will be quite different, thus affecting the starting transient of the pipe, and considerably altering its tonal effect.

Languid and Upper Lip

The most sensitive adjustment in voicing is the height of the languid, with reference to the lower lip. A few hundreths of a millimeter will make a noticeable difference to the speech of the pipe. The main effect is in the initial speech; to raise the languid is to make the pipe slow in speaking and to lower it produces the reverse effect.

There is evidently a connection between the speed of speech and the direction of the windstream relative to the upper lip; since much the same effect on the speed of speech can be made by altering the upper lip in the appropriate direction. The two effects cannot, from the organbuilder's viewpoint, be regarded as interchangeable, since there is evidently an optimum setting of the upper lip.

Condition of Upper Lip

The upper lip may have its edge beveled in varying degrees, or rounded by burnishing. These changes make a very slight difference to the overtones and the noise, but the final effect is apparent only in a whole stop of pipes.

A more effective alteration is to cover the upper lip with a thin layer of leather. This reduces the higher overtones considerably, and also makes it possible to use a higher wind pressure. The method needs very careful handling, since it is difficult to avoid a dull tone which does not blend easily.

VARIETIES OF TONE

It is now possible to establish some reason for the wide variety, both of tone and of construction, found in organ pipes.

Open Pipes

Consider as an example an attempt to increase the power of a particular diapason pipe. An obvious approach is to increase the wind supply by opening the bore; this, however, will have the effect of making the tone keener as well as louder. If this process is continued, the pipe will "overblow" to its octave, and will no longer sound its fundamental. Cutting the mouth up higher will make it possible to restore the tone, approximately, to its original quality, but with an increase in intensity.

For still more powerful tone a similar pipe, but of larger diameter, may be used. There is, however, a fairly definite aesthetic limit to such a process, since when a diapason pipe is voiced too loudly its tone sounds "forced" or "coarse." It would be instructive to correlate this subjective judgment with a tonal analysis of the pipe.

Since cutting up a pipe without increasing the wind supply reduces its harmonic development, it might be expected that a diapason could by such means be transformed into a flute; and indeed it is possible to produce flute tone in this way. As a rule, a pipe designed as a flute will be of larger scale than a diapason; but a high mouth, sometimes arched, is a more usual feature.

For string tone, on the other hand, the influence of the diameter is of much greater importance. Early string-toned pipes were characterised by a narrow scale, low mouth, and fine nicking. From one point of view, they may be considered as diapasons with the voicing carried to extremes, and this caused them to be somewhat unstable, and slow in articulation. The type of string voicing known today was introduced in the 1870's; the pipes were of very narrow scale, with the bore so large that the pipe would overblow to its octave. A roller-bridge (a small-diameter rod) was then fitted in front of the mouth, which had the effect of restoring the frequency to the fundamental of the pipe. This made possible a promptness of speech and a quality of tone unknown before.

The phenomenon of overblowing to the octave is employed in the case of the "harmonic flute" and assisted by boring a small hole at the center of the pipe-body. There is then a pressure node at this point, and the pipe-body thus becomes one wavelength long.

From the foregoing, it will be seen that there are three main variables—scale, mouth-height, and quantity of wind. J. G. Töpfer, a German organist and professor of music, made experiments with flue pipes to establish some relationship.[20] Putting L, D, A, and M as length, diameter, and mouth-area of pipe, and quantity of wind, he obtained two coefficients n and r, such that

$$n = ML^{\frac{1}{2}}D^{-2} \quad \text{and} \quad r = ML^{\frac{1}{2}}A^{-1}.$$

The coefficients are not dimensionless, and vary somewhat with the pitch of the pipe. They are constant for one power and one tone-quality. The range over which they hold is not known; clearly they cannot be used to compare pipes differing widely in construction, but they do indicate approximately the influence of the different factors on the tones of pipes.[21]

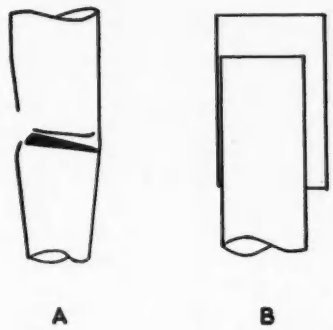

FIG. 6. (A) Diagram of mouth of double languid pipe; (B) "compensator-amplifier" fitted to top of pipe.

Returning to the diapason pipe voiced for great power, two modifications introduced by the Willis firm should be given. The first is the "double-languid" pipe (Fig. 6A). In this the airstream from the flue entrains air flowing between the languid and the upper "false" languid, which assists in the production of more powerful tone. Another is the addition of a short length of tubing to the open end of the pipe (Fig. 6B). This is known as a "compensator-amplifier" and enables the pipe to be voiced on higher pressure while maintaining good tone; it is also of value at moderate pressures. A detailed theoretical treatment of this would be valuable.

Two papers have been published dealing with modifications of the tone of open pipes by voicing them. Kuhn[22] had a pipe made up with all the parts adjustable, and measured the intensities of the first six harmonics with different

[20] J. G. Töpfer, *Die Theorie und Praxis des Orgelbaues*, herausg. von M. Allihn (Voigt, Weimar 1888). The "Theory" described here was published in 1833.

[21] F. E. Robertson, *A Practical Treatise on Organ Building* (Sampson Low, London 1898). This contains an account of Töpfer's theories and their application.

[22] K. T. Kuhn, Ph.D. dissertation, Berlin, 1939.

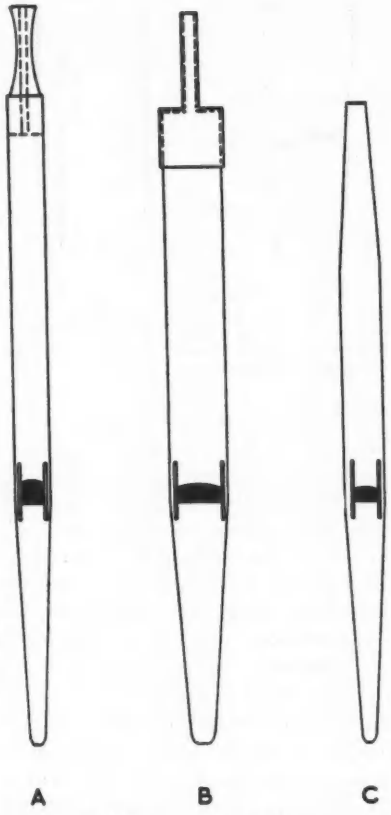

FIG. 7. Typical stopped pipes (middle C); (A) lieblich gedackt, (B) rohrflöte, (C) spillflöte.

settings. Ingerslev and Frobenius[23] used altogether 17 pipes of the same pitch but differing in diameter and mouth dimensions. The results of the investigations cannot be summarized simply; the latter authors tested Töpfer's formulas, and found that the agreement was only fair. Kuhn's conclusions may be affected by the fact that his pipes differed so widely from normal constructions. Ingerslev and Frobenius carefully voiced each pipe to speak properly.

Stopped Pipes

A cynic might remark that the differing behavior of the air-column standing waves in stopped and open pipes is the only point satisfactorily explained by physical theory. Even here the end corrections are uncertain, but there are several points which are in fact explicable by theory.

The early type of stopped pipe was a wide-scaled pipe, giving a comparatively dull tone. Schulze introduced the "lieblich gedackt" in 1851, a stop of much brighter tone, which made a tremendous impression. It was a much narrower scaled pipe; later examples frequently had a hole through the stopper, thus encouraging some of the even harmonics (Fig. 7A). A peculiarity in the tone (and evidently a factor in its charm) is a pronounced initial transient, which is sometimes inharmonic. A typical transient is shown in Fig. 3C.

The hole through the stopper implies that there is another resonating length in the pipe. This principle is used in the rohrflöte, Fig. 8B, a stop invented during the 16th century. The theory of this was worked out in 1884 by Gerhardt,[24] and has been discussed more recently.[25] In general, theory and practice seem to agree well, but a full examination has not yet been made.

Many other constructions which may be called half-stopped, such as those in Fig. 7C, were widely used in the classical period and are being deservedly revived today. They should provide interesting physical problems. Somewhat similar in principle is the Haskell pipe, a construction used for bass pipes when space is limited. Jones tested two pipes and found excellent agreement between the theoretical and examined behavior.[26]

Voicing and Final Treatment

The sequence of events leading up to the satisfactory installation of a pipe in a finished organ may now be described. The process begins (or should begin) when the builder visits the building in which the instrument is to be installed and judges its acoustic properties. From pre-existing knowledge of the approximate size of the organ he knows the tone and the degree of power which is required from the particular stop, and from experience he can decide approximately on the wind pressure, scale, and dimensions of mouth.

[23] F. Ingerslev and W. Frobenius, Trans. Danish Acad. Tech. Sci. No. 1, 7 (1947).

[24] R. Gerhardt, Nova Acta der Kaiserl. Leop.-Carol.-Deutschen Akademie der Naturforscher **47**, 5 (1884).

[25] Nature **114**, 309, 465, 573, 787 (1924).

[26] A. T. Jones, J. Acoust. Soc. Am. **8**, 196 (1937).

The pipe is then made in the metal shop. The following further factors are decided here—material, wall thickness, length of foot, angle of bevel of languid, shaping of lower lip. The scale-ratio throughout the stop is also decided.

All the pipes of the stop come then to the voicing shop. The voicing procedure of one leading English builder is as follows: The pipes are cut to length, bores opened to required diameters according to a prearranged set of measurements, and cut up; the upper lips are then beveled, nicking carried out and width of flue adjusted. The pipes are then ready to be placed on the voicing machine and brought to correct speech, tuned, regulated and tuned again.

When this work is finished, the pipes are ready to be installed in the finished organ. Here, owing to the different acoustic environment, they may appear to speak in a different fashion from that in the voicing shop. Final "regulation"—mainly minor alterations of loudness made by adjusting the bores—is carried out very carefully, listening from a representative place in the building.

ORGAN PIPES IN COMBINATION

Blend

If two organ pipes an octave apart are sounding, their tones will not necessarily blend. The conditions under which they do, and the precise tonal qualities required, would provide an instructive study. It appears however that adequate harmonic development of the lower-pitched pipe is the principal requisite; thus the harmonics of the higher-pitched pipe reinforce tones already existing. When two properly blending pipes are sounding, it is impossible to distinguish them separately.

Much use is made in organs of pipes tuned to the higher harmonics of the unison. "Synthetics" are flute-toned pipes which blend with the unison giving a new tone-color, but lose their own identity. Similar stops of diapason tone, while also creating a new tone-quality, remain to some extent separately audible. Such "mutation" stops were widely used in the classical period; they are important since they make possible tone colors which can be produced only on the organ.

Scale-Ratio

Consider two pipes of the same stop, an octave apart. The mouths are usually geometrically similar. From earlier considerations it will be realised that tone and power depend on both diameter and size of bore, and the diameter of the upper pipe relative to the lower must be very carefully chosen so that its tone and power match the lower one. If, say, the diameter of the upper pipe is too small, it will be too keen if of the same power, and not loud enough if of the right tone. Such an effect, if excessive, would cause chords played on such a stop to sound unbalanced, or would affect melodic phrases. Some degree of change in quality and loudness from bass to treble is, however, desirable. A somewhat rich lower range and a bright and adequately powerful treble is a combination often used.

Töpfer "derived" the scale-ratio of $1:\sqrt{8}$, meaning that two pipes an octave apart should have their cross-sectional areas in this ratio. His reasoning leading to this was somewhat dubious; it was, in fact, so chosen to make his coefficients n and r independent of pitch. Ratios in use today approximate to this value, but vary for different types of stop. The whole balance of the tone, and much of the effect of an organ, depends on the correct scaling, and it requires considerable artistic judgment to make it satisfactory.

Ensemble

One of the most important features of an organ is the "full organ" effect, the ensemble when all the stops are drawn. In practice only a selection of the more powerful stops, forming "choruses" of considerable power, are used.

The concept of a chorus is fundamental to an organ of any size; it represents the extension of the unison tone upwards and downwards by several octaves, including stops representing off-unison harmonics of the fundamental. The physical and physiological reasons why the ear should prefer such an array of tones is not known; possibly there is a preference for the acoustic energy to be spread over a considerable frequency range, but whatever the reason may be there is no question of the aesthetic success of

the method. In the last century Hope-Jones built instruments with dull-toned leathered diapasons, intending additional unison ranks of high harmonic development to provide the desired effect; but in spite of the excellence of the individual stops such ensembles become most wearying to listen to for any length of time.

The diapason chorus only will be mentioned here; while it is not as powerful as the reed chorus, it is certainly the most characteristic of the organ. The prerequisite for a successful chorus stop is, as stated above, sufficient harmonic development. (Many choruses with a unison diapason of very powerful but dull tone give an appearance of blending, but the reason is apparently that the powerful fundamental tone swamps everything else.)

The smallest diapason chorus which can reasonably be considered complete would contain one or two unison diapasons, a sub-unison, an octave and superoctave, and ranks of pipes sounding the 3rd, 5th, 6th, and 8th harmonics of the unison. The variations of such a scheme appear inexhaustible, and can themselves account for the difference in tone between organs by different builders. Some of the salient points may be outlined. The sub-unison rank is often given a higher degree of harmonic development than the diapasons; the unison rank is the most powerful, but the degree to which the higher pitched ranks decrease in power varies greatly between builders. The octave rank differs slightly in voicing from the unison, being appreciably brighter; the off-unison ranks may have narrower mouths, to subordinate their tone somewhat. Together with this is the whole policy of the variation in tone and power of the stops throughout the key compass. In fact, the highest degree of skill and experience is needed in the construction of a successful chorus, and very many instruments are unsatisfactory in this respect.

The composition of the principal diapason chorus in this country's largest organ, at Liverpool Cathedral, is as follows: a 32-ft violone (a stringy diapason), 16-ft diapason, five unison diapasons (of different timbre and power) three octaves and two superoctaves. Together with these there are stops giving the 3rd harmonic of the 32-ft, the 3rd and 5th harmonics of the 16-ft, and eleven (some duplicated) ranks representing the 3rd, 5th, 6th, 7th, 8th, 10th, 12th, 16th harmonics of the unison. The aural effect of even a single note played on such a magnificent chorus is that of a complete and rich mosaic of tone from bass to treble; the ear clearly assigns the unison pitch to the sound, the octaves and off-unisons blend without clashing, and the whole effect is one of balance and homogeneity over the whole range of tone.

CONCLUSION AND ACKNOWLEDGMENTS

It will be realised that much of the research on organ pipes must lie in the rather dim borderlands; part of it is between physics and aesthetics, much of it an investigation of the traditions of a craft, while the work on aerial vibrations infringes on aerodynamics. These facts probably account for much of the fascination which the author, at any rate, finds in this study.

This is perhaps also the reason why the physicist who attempts such work without consulting with an organ-builder is risking the waste of a good deal of time on side issues. The author is pleased to take this opportunity to acknowledge the considerable assistance which Mr. Henry Willis has given to him over the practice of dealing with pipes. Mr. Willis has also read through this article and made several corrections and suggestions; but the author takes full responsibility for the statements made.

Grateful acknowledgement is also made to Professor A. M. Taylor for his continued interest and encouragement.

Practical Aids for Physics Teachers

The *American Journal of Physics* plans to introduce a new feature as soon as possible under the title, "Practical Aids for Physics Teachers." At first, it may begin in a small way, but we hope that it will grow to occupy several pages in every monthly issue.

The aids that go under such a title must be contributed; they cannot be prepared in the Editor's office. A list of possible aids and some suggestions for the contributor were published in the April issue (p. 266).

Please send me contributions. The only stipulation that I must make is that *everything* must be typed double-spaced. Otherwise there is no room for editorial markings. A mimeographed copy that is already double-spaced and of good quality would be acceptable.

This is your Journal, your Association. Won't you help?

THOMAS H. OSGOOD
Editor

NOTES AND DISCUSSION

The Ideal Gas Equation

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A STRICT derivation of the ideal gas equation was given by H. Poincaré based on the two conditions defining an ideal gas: (a) the gas obeys Boyle's law, $Pv=C(T)$, and (b) the internal energy U is solely a function of temperature $U=f(T)$. It is the purpose of this paper to give another and what seems to us a more straightforward derivation based on the same definition.

However, since Poincaré's derivation does not seem to be prominent in the American literature it may not be out of place to give it here.

Poincaré started with the combined first and second law in the form

$$dS=dU/T+(P/T)dv, \tag{1}$$

where S is the entropy of the system.

Now since $U=f(T)$, we have

$$dU=f'(T)dT,$$

which when combined with Eq. (1) yields

$$dS=(f'(T)/T)dT+(P/T)dv. \tag{2}$$

Since dS is an exact differential, Eq. (2) gives

$$\frac{\partial}{\partial v}\left(\frac{f'(T)}{T}\right)=\frac{\partial}{\partial T}\left(\frac{P}{T}\right)=0.$$

The net result is that P/T is simply a function of volume only, that is,

$$P/T=g(v). \tag{3}$$

Now if we eliminate the P between Eq. (3) and $Pv=C(T)$ we get

$$vg(v)=C(T)/T.$$

Because the left-hand side of this last equation contains no T it is necessary that T does not appear on the right-hand side. This can only be if $C(T)=RT$, and Poincaré's proof is complete.

The present derivation starts with the so-called energy equation

$$\left(\frac{\partial U}{\partial v}\right)_T=T\left(\frac{\partial P}{\partial T}\right)_v-P=0, \tag{4}$$

where we have already employed condition (b) of the definition of an ideal gas.

If now we substitute Boyle's law in Eq. (4) we get

$$\frac{T}{v}\frac{dC(T)}{dT}-\frac{C(T)}{v}=0, \tag{5}$$

where it is important to note that $dC(T)/dT$ is a total derivative. Equation (5) can be rewritten as

$$\frac{dC(T)}{C(T)}=\frac{dT}{T}. \tag{6}$$

The solution of Eq. (6) readily yields that $C(T)=RT$ so that we finally get

$$Pv=RT.$$

On the Resolving Power of a Prism

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IN this note the authors propose to discuss the resolving power of a prism in positions other than minimum deviation.

Consider the passage of a plane wave through a principal section (supposed to be an isosceles triangle) of a prism, whose refracting angle is A. Let i_1 and i_2 be the angles of incidence and emergence and r_1 and r_2 the corresponding angles of refraction within the prism. If θ is the total deviation produced by the prism, and a the width of the aperture of the emergent wavefront, the dispersion and resolving power R are given by

$$\frac{d\theta}{d\lambda}=\frac{\sin A}{\cos i_2\cos r_1}\frac{d\mu}{d\lambda}, \tag{1}$$

and

$$R=a\frac{d\theta}{d\lambda}. \tag{2}$$

For the calculation of aperture we have to distinguish between the two cases when the rays undergo less deviation or greater deviation than those traversing the prism symmetrically.

(1) In the first case $r_1>A/2$ and it can be easily seen that

$$a=(l\cos i_2\cos r_1)/\cos r_2, \tag{3A}$$

where l is the side of the prism.

(2) In the second case $r_1<A/2$, and

$$a=l\cos i_2. \tag{3B}$$

The expressions for resolving powers R can be obtained from Eqs. (1), (2), (3A) and (3B). Thus when $r_1>A/2$,

$$R=\frac{l\sin A}{\cos r_2}\frac{d\mu}{d\lambda}=t\frac{d\mu}{d\lambda}\cdot\frac{\cos\tfrac{1}{2}A}{\cos(A-r_1)}=R_0\frac{\cos\tfrac{1}{2}A}{\cos(A-r_1)}; \tag{4A}$$

and when $r_1<A/2$,

$$R=\frac{l\sin A}{\cos r_1}\frac{d\mu}{d\lambda}=l\frac{d\mu}{d\lambda}\frac{\cos\tfrac{1}{2}A}{\cos r_1}=R_0\frac{\cos\tfrac{1}{2}A}{\cos r_1}, \tag{4B}$$

where t is the base of the prism and $R_0=td\mu/d\lambda$ is the resolving power of the prism in the minimum deviation position.

TABLE I.

γ_1	0°	10°	20°	30°	40°	50°	60°
R/R_0	0.8660	0.8792	0.9219	1	0.9219	0.8792	0.8660

Equations (4A) and (4B) can be written as

$$R = t' d\mu/d\lambda, \tag{5}$$

where t' may be called the effective base of the prism and is the longest path travelled by the light in the prism.

It is evident that the resolving power varies with the angle of incidence, being maximum in the minimum deviation position ($r_1 = r_2 = A/2$) and minimum when $r_1 = 0$ or A, i.e., for normal incidence or emergence. Thus $R_{\min} = R_0 \cos A/2$.

The variation of R/R_0 with r_1 for a 60° prism is given in Table I and diagramatically illustrated in Fig. 1.

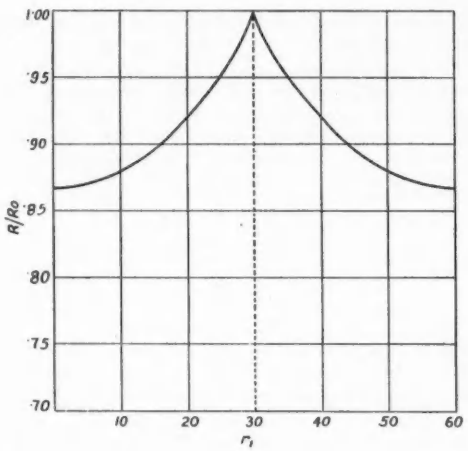

FIG. 1. Variation of the ratio R/R_0 with r_1, where R is the resolving power of the prism, R_0 its resolving power in position of minimum deviation, and r_1 the first angle of refraction within the prism. The curve is drawn for a 60° prism.

If, however, the angle of incidence is kept constant the resolving power varies with wavelength in a spectrum. The variation due to the factor $\cos\frac{1}{2}A/\cos(r_1 \text{ or } r_2)$ is small, as r_1 and r_2 do not change appreciably with wavelength. In such a case the dispersive power $d\mu/d\lambda$ is mainly responsible for the variation of resolving power. Using Hartmann's dispersion formula we can write, for $r_1 > A/2$,

$$R = \frac{t \cos\tfrac{1}{2}A\{-c/(\lambda-\lambda_0)^2\}}{\cos\left[A - \sin^{-1}\left\{\dfrac{\sin i_1}{\mu_0 + c/(\lambda-\lambda_0)}\right\}\right]}; \tag{6A}$$

and for $r_1 < A/2$,

$$R = \frac{t \cos\tfrac{1}{2}A\{-c/(\lambda-\lambda_0)^2\}}{\left(1 - \dfrac{\sin^2 i_1}{\{\mu_0 + c/(\lambda-\lambda_0)\}^2}\right)^{\frac{1}{2}}}. \tag{6B}$$

Group-Phase Velocity Demonstrator

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THE speed with which the concepts of group velocity, phase velocity, and beats are grasped, integrated, and retained can be materially aided with the following simple demonstration apparatus.

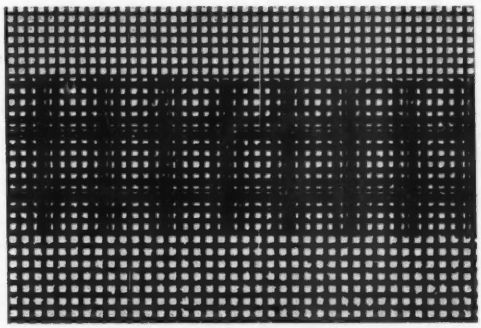

FIG. 1. Fifteen-mesh overlay pattern obtained by overlapping pieces of 65- and 80-mesh screening (magnification approximately 5 times).

Two pieces of Lectromesh screen (an electrolytically deposited screen made by the C. O. Jelliff Manufacturing Corporation, Southport, Connecticut) of dimensions 2 in. × 10 in. and of 65 and 80 mesh, respectively, are cut along the mesh and are separately mounted between pieces of cleared 2 in. × 10 in. spectrographic plates. The screening should be mounted parallel to one long edge of the glass plate, and each sandwich should be bound like a lantern slide with care taken so that the binding is parallel to the screen. Finally fiducial marks may be scribed or ruled in corresponding locations on each of the two sandwiches.

When the completed sandwiches are overlapped so that, say, an inch of 65-mesh screening, an inch of overlay, and an inch of 80-mesh screening are visible, the overlay displays a 15 mesh pattern (Fig. 1). Holding the 80-mesh screen stationary, a displacement of the 65-mesh screen to the right causes the 15-mesh pattern to move to the left. Holding the 65-mesh screen stationary, a displacement of the 80-mesh screen to the right causes the 15-mesh pattern to move to the right with greater speed than that of the 80-mesh screen. By superimposing a uniform velocity for the assembly as a whole upon the relative motions of the two screens, the difference between group and phase velocity can be made clear. The overlay 15-mesh pattern may be identified as the group wave while the 65- and 80-mesh screens may be considered as the separate phases. Thus the group velocity is distinctly different from the velocity of either phase, and the concept of group velocity is meaningful only when the two phases move with different velocities, hence in a dispersive medium.

The same apparatus may be used to illustrate beats by moving the two screens at the same speed. Here the

"group velocity" of the beat is the same as the separate phase velocities, corresponding to audible beats in air, an essentially nondispersive medium.

A Dissipation Factor Anomaly

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IN studying capacitors the question arises as to whether the resistive loss which is present should be considered as a resistance in series or in parallel with the capacitance. It has been found that the dissipation factor is independent of the model used. In working on natural cork with a high moisture content in connection with this problem an anomaly has been found.

The rapid growth of electronics with circuits using inductors and capacitors has led to increased use of these terms:

storage factor (figure of merit) as applied to an inductor $Q=X/R=\omega L/R=\tan\theta$

dissipation factor as applied to a capacitor $D=R/X=\omega CR=\cot\theta$

where X=reactance; R=resistance; C=capacitance; L=inductance; $\omega=2\pi$(frequency); θ=angle between the directions of X and R. For small values of D, where $\cot\theta=\cos\theta$, the term "power factor" is used.[1]

In each case R represents what is often an undesirable loss. An ideal inductor would have $R=0$, making Q infinite. An ideal capacitor would have $D=0$. For capacitors the question arises as to whether the R which represents the loss (in this case much smaller than for inductors) is equivalent to an R in series, in parallel, or to some other combination.

Crosbie[2] assumed three models: an R in series with a C; an R in parallel with a C; and an R in series with a C and R in parallel. He then proved that the working equation is the same regardless of the model. Consequently, using data from a capacitance bridge he obtained the same numerical value for D for each model. This leads to an interesting conclusion: D is an important measurable electrical quantity which does not depend on a model. The calculation for dielectric constant K depends entirely on the model.

In general a low value of D is desired, but today, with dielectric heating and conductive rubber, studies have been made of large D's. Most substances show a steady increase of D as K increases. A careful study of natural cork containing various percentages of water by weight, showed a maximum D at 7 percent moisture using a frequency of 1 kc/sec. At the same time K showed a steady increase.

Table I shows the variation of moisture with D, K, G (admittance), and B (susceptance), where K, G, and B were calculated assuming a parallel model. For the capacitor with no dielectric G was 0, B, 7.4×10^{4}.

Careful checks were made with the apparatus—a Schering bridge; readings were taken in a few seconds; but the effect still remained. Similar results were obtained using a moisture register[3] with insulating firebrick.

In each case where a reversal of D occurred, K increased steadily, regardless of the model. Nothing has been found in the theory to lead one to expect this reversal of D.

Since these data were taken over a period of a year, it was not possible to study the effect at other frequencies, except for dry natural cork. Indications are that similar results would have been obtained.

TABLE I. Properties of cork.

Percent Water	D	K	G	B
0	0.0076	1.27	0.072×10^{4}	9.4×10^{4}
2	0.013	1.40	0.135	10.4
4	0.125	1.60	1.46	11.7
6	0.890	3.27	12.0	13.5
7	1.200	6.05	22.1	18.4
8	0.870	10.2	37.3	42.9
9	0.605	21.5	70.7	116.9
10	0.445	28.3	78.9	175.2
12	0.305	33.6	69.0	227.0

[1] ASTM Standards D150–44T.
[2] E. A. Crosbie, "Schering bridge measurements of high loss dielectric materials." Unpublished Master's thesis, Washington and Jefferson College Library, 1948.
[3] Moisture Register Company, Alhambra, California.

Striations in Electromagnetic Stationary Waves

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IN the Lecher-wire method of measuring the wavelength of stationary electromagnetic waves, using a pair of parallel wires, a short-circuiting bar with a small incandescent lamp in it is usually used to determine the positions of the nodes and loops.[1] A much more striking and effective class demonstration is to use a fluorescent lamp placed between the wires as illustrated in Fig. 1. One of the 8-foot tubes is about the right length for our oscillator, which has a wavelength that is variable (tunable) from about 3 to 4 feet. The lamp lights except at the potential nodes, and

FIG. 1. Luminosity in a fluorescent lamp placed between Lecher wires.

FIG. 2. Periodic character of the luminosity in a fluorescent lamp placed between Lecher wires.

changes in wavelength and coupling are easily demonstrated. For quantitative determination of wavelengths, the incandescent lamp is more precise.

However, the phenomenon that I wish to report is entirely new to me. Instead of a continuous light column between the dark potenial nodes, there is a definite pattern of striations or alternate light and dark bands which appear all along the lighted portions of the tube, part of which is shown in Fig. 2. These bands are about 4 or 5 cm apart and are an entirely separate phenomenon from the dark regions at the voltage nodes. The behavior of these striations is surprisingly like that of the little ridges that appear in the dust patterns of stationary sound waves, using the Kundt's tube method.[2]

I am wondering whether this phenomenon has been reported before in stationary electromagnetic waves, and am interested in an explanation. Can it be a phenomenon of all stationary waves? Since striations have been known in discharges through gases at reduced pressure for many years it is more likely a related phenomonon.[3]

[1] Richard M. Sutton, *Demonstration Experiments in Physics* (McGraw-Hill Book Company, Inc., New York, 1938), No. A-37, p. 455.
[2] Andrade, Trans. Roy. Soc. (London) **A230**, 413–445 (1932). Seifert, Am. J. Phys. **7**, 421 (1939).
[3] T. Donahue and G. H. Dieke, Phys. Rev. **81**, 248 (1951).

AMERICAN JOURNAL OF PHYSICS VOLUME 21, NUMBER 5 MAY, 1953

LETTERS TO THE EDITOR

Need for More and Better Trained High School Physics Teachers

A GREAT deal has been written during the past few years emphasizing the scientific manpower shortage[1] and the need for encouraging able and qualified students to make careers in the sciences, especially in industrial and governmental research.[2]

It seems to me that one very important matter is being overlooked: namely, the development of more and better physics teachers, especially for the high schools. One hears on every hand that high school and college physics teachers are leaving their schools and taking positions in government work and industry. Low salaries, heavy teaching loads, a desire for better opportunity for research, as well as a genuine desire to contribute more to the defense effort may prompt many teachers to leave their positions. Those who remain have heavier teaching loads and give perhaps poorer instruction. Furthermore, it has been my experience that very few physics majors in college even consider teaching as a profession, particularly on the secondary-school level. This may be due in part to the fact that very little effort has been made to point out to the physics students some of the advantages and richly rewarding experiences (except financial) of the teaching profession, such as helping young minds to develop, more individual freedom in one's work, and congenial associates. The important questions, however, are "Who is going to teach and stimulate interest in prospective young scientists?" and "How are these teachers to be recruited and trained?"

I believe there is a need to restudy the training given to high school physics teachers. I have made no special study of this question and do not qualify as an educational specialist, but from my limited experience with requirements for high school physics teachers, it often appears that either so many courses in physics are required that a student is discouraged from entering the field or so few are required that the teacher is not adequately prepared. Furthermore, for those who have little training in physics as undergraduates there is often very limited opportunity to obtain further training in graduate schools.

Realizing the need for special graduate study for teachers with limited training in the physical sciences which would lead to a master's degree and which would insure the teacher an increase in salary, the deans of the Graduate School and the Teachers College and the heads of the physics and chemistry department at Marshall College have conferred and have decided to embark on an experimental program of graduate courses in physics, chemistry, and education leading to a Master of Arts Degree in Eduaction with a major either in physics or chemistry. The physics courses offered are designed to help the teacher to present more effectively the basic principles, methods of development, and the subject matter of physics, to extend his own knowledge of the fundamentals of physics, and to bring him up to date in its modern applications, as for instance, in electronics and nuclear physics. At present, for the convenience of the teachers, these courses are offered primarily in the summer sessions of the College. The physics and chemistry departments also plan to cooperate with the high schools by offering some refresher workshop-type courses and by faculty members giving demonstration lectures at high school assembly programs and talks to science classes designed to interest the pupils in making careers in science, including teaching.

I have found through correspondence that several

colleges in different sections of the country are experimenting with similar graduate programs in the physical sciences, and I should like to suggest that the officers or some committee of the American Association of Physics Teachers might investigate further what is being done in the way of graduate programs designed especially for high school physics teachers and also what is being done in the way of recruiting students to become physics teachers.

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[1] C. A. Mills, Science **116**, 601 (1952).
[2] Allen V. Astin, Sci. Teacher **XIX**, 257 (1952).

Observations on the Smoke Trail of a Sky-Writer

WHILE driving on the highway recently my attention was drawn to a sky-writer whose trail fell between me and the sun and less, perhaps, than 30 degrees above the horizon. On first observation I had the feeling that the sunlight reaching me *through* the smoke trail was reddish-orange in color, and a moment's study confirmed the notion that the smoke possessed a predominantly reddish grey hue. Within the next several seconds my own course changed my line of sight so that I was now seeing the smoke side on, that is, not along the direction of the sunlight but nearly normal to it. At this moment the smoke looked its natural bluish-grey. This observation struck me as an excellent illustration of scattering, and it might well be called to the attention of students. It is obvious that the observer need not change his position; all one needs is that the sky-writer should pass between him and the sun.

I am led to suggest also that some interesting observations on wind direction and velocity may be made on such smoke trails.

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A Problem in Electrostatics

MANY college textbooks which cover the fundamentals of electrostatics contain a discussion of the following experiment. A large hollow conducting sphere with a small opening in its side has a charge q. A small conducting sphere with a charge q' is brought from infinity and inserted through the opening into the larger sphere. The two spheres are then brought in electrical contact. A discussion of the potential and charge-distribution changes which result seems to prove very enlightening to students and paves the way for a discussion of the principle of the Van de Graaff generator.

A few years ago a student asked a question which seemed to involve a paradox, and which caused so much class discussion and aroused so much interest, that the author has used it in all subsequent classes on the subject.

Suppose the now uncharged small sphere is removed from the larger sphere and carried to infinity. The total energy of the system is the same as when the uncharged body was inside the larger sphere. Yet, because of the charge induced on the small sphere when it is outside of, but near to, the larger sphere, energy is required to withdraw it from this position to infinity. How can this be reconciled with the law of conservation of energy?

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Concerning the Frequencies Resulting from Distortion

THIS note concerns the question raised recently by Maxwell[1] concerning the physical origin of the harmonic frequencies which appear in the analysis of a rectifier circuit.

Van Name has suggested[2] that these frequencies are not physically present at all. If this were the case, it would not be possible to operate an harmonic analyzer, which is essentially a tuneable highly selective amplifier and voltmeter. As a matter of fact, electronics-laboratory courses frequently include an experiment in which the amplitudes of the harmonics of a nonsinusoidal wave are measured and compared with those values calculated by a Fourier analysis.[3]

Thus experiment shows that these frequencies are physically real and present in the system. They have been generated by the nonlinear response characteristic of the rectifier itself. In general, if one is given the input wave form and the characteristic of a nonlinear device, one may calculate the amplitudes and phases of the harmonics which will be generated. The simplest case is probably that of the amplifier with a quadratic plate-current characteristic and a sinusoidal input signal; that is, $i_p(e_g)=G_1e_g+G_2e_g^2$, and $e_g=E\sin\omega t$ in the usual notation. The analysis of the rectifier is not quite so simple but it can in principle be carried out.

There are other instances of frequency generation by nonlinear devices. It is perhaps not generally appreciated that the phenomenon of combination tones (due to the superposition of frequencies whose sum or difference is an audible tone) is due to the nonlinear response of the ear as a detector. If the experiment is carried out with tuning forks of frequencies f_1 and f_2, one finds that a fork of frequency (f_1-f_2) is *not* sympathetically excited although one may hear the combination tone. There is *no energy* present in the combination frequency as long as the oscillations occur only in linear media. It is only in the ear, a nonlinear device, that the combination frequency occurs.[4]

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[1] Howard N. Maxwell, Am. J. Phys. **20**, 310 (1952).
[2] F. W. Van Name, Jr., Am. J. Phys. **20**, 520 (1952).
[3] For another experiment see E. H. Schulz and L. T. Anderson, *Experiments in Electronics and Communication Engineering* (Harper and Brothers, New York, 1943), p. 101.
[4] F. W. Sears, *Principles of Physics* (Addison-Wesley Press, Cambridge, 1947), Vol. I, p. 502. R. R. Ramsey, Am. J. Phys. **8**, 237 (1940). *Fundamentals of Acoustics*, Kinsler and Frey (John Wiley and Sons, Inc., New York, 1950), p. 383.

Exciting a Kundt's Tube with a Siren

THE remarks of Baez[1] on the Kundt's tube prompt me to add that the particle motion is made more dramatic when the tube is excited with a siren. Also the accuracy of the velocity of sound measurements is improved when this method is employed.

What may be described as cartwheels of cork dust are formed at the center of each antinode when the tube is excited in this manner. This is accomplished by adjusting the pressure and angle of the air jet. A remarkable pattern of particle motion is observed throughout the volume of the tube.

These cartwheels of dust are very thin (0.5 mm) and rotate slowly in dynamically stable motion. Thus the measurement of the wavelength is made very accurate. The frequency as determined with a siren is also more accurate than the dial readings on the usual audio oscillator. The velocity of sound can readily be determined within 1 percent of the accepted value.

The tube employed in this experiment was a glass cylinder 2.5 cm in diameter and 125 cm long. One end of the tube was closed. The siren end was covered with a card having a one-quarter inch hole. The position of the hole was adjusted with respect to the air jet to produce maximum effects within the tube.

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[1] Albert V. Baez, Am. J. Phys. 21, 64 (1953).

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ANNOUNCEMENTS AND NEWS

Book Reviews

The Classical Theory of Fields. L. LANDAU AND E. LIFSCHITZ. Translated from the Russian by Morton Hamermesh. Pp. 354+ix, 16×25 cm. Addison-Wesley Press, Cambridge, Massachusetts, 1951. Price $7.50

This is an interesting publication and will no doubt find its way into the libraries of many students of physics. The fundamentals of the mathematical techniques of field theory are well presented, so that those who are interested in this branch of physics will be able to obtain the basic concepts and techniques without having to depend heavily upon the lectures by experts and upon the original papers.

Since the subject of electrodynamics is presented from the field-theoretical point of view, this book will serve admirably as a text for an introductory course in field theory. The content and the notation are such that there will be very little difficulty in making transition to quantum field theory. However, it is rather doubtful that this book will be appropriate as an introduction to electrodynamics. In the conventional development of this subject, the experimental and the historical approach is used, i.e., the field equations are deduced from the observations by Ampere, Faraday, and others. It seems to the reviewer that the latter approach is more meaningful to a beginning student; after he has acquired a good foundation in *physical* electrodynamics, he will be able to appreciate the elegance of *formal* electrodynamics.

Also, the value of this book as a reference would have been increased manyfold had the authors indicated a few more references to original papers and other books dealing with other aspects of field theory. For example, the section on the energy-momentum tensor reminds one of a paper by W. Pauli;[1] the section on the characteristic vibrations of the field reminds one of the treatment of the classical electromagnetic field by W. Heitler.[2] References to such publications would have made it easier for students to go on to make a more nearly exhaustive study of field theory, be it classical or quantum, after having mastered the contents of this book.

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[2] W. Heitler, *The Quantum Theory of Radiation* (Clarendon Press, Oxford, 1936).

An Introduction to Scientific Research. E. BRIGHT WILSON, JR. Pp. 375+xiii, Figs. 53, 15.5×23.5 cm. McGraw-Hill Book Company, Inc., New York, 1952. Price $6.00.

If it requires great courage on the part of a scientist to write a treatise about his research specialty (and I am sure it does), how much greater must his courage be to attempt a treatise on how to do research in general! Such a hero or martyr (for indeed he must be both) has already put his feet and hands in the holes of the pillory and invited his enemies to snap shut the padlock!

Fortunately I am no enemy of the author of *An Introduction to Scientific Research* and indeed I feel the urge immediately to express my deep admiration for his brilliant achievement in this well arranged, well conceived and executed work of 365 pages. The explicit object of the book is "to attempt to collect in one place and to explain as simply as possible a number of general principles, techniques, and guides for procedure which successful investigators in various fields of science have found helpful." The book is stated to be "specifically intended for students be-

ginning research and for those more experienced research workers who wish an introduction to various topics which were not included in their training."

I feel that the value of the book will be far greater for the last-named class than for beginners. In my opinion, its most outstanding feature is that it aims to acquaint scientists in general with the valuable and vital new field of mathematical statistics about which so many of us are so blissfully (nay woefully!) ignorant.

Time was, in the memory of this reviewer, when the most eminent research physicist of his day summed up his pedagogical theory as regards instruction in research in physics by saying, "Throw them in the laboratory and let them sink or swim!" Many of them sank, but then "there were giants in those days" too. A somewhat younger theoretical physicist, who in the sequel achieved easily comparable eminence also made the much quoted remark of that era, "You don't *have* to be crazy to do physics, but it helps." Nowadays, with the world split into two mortally antagonistic camps, we are coming to realize that most acute of all may be the shortage of highly trained scientific research talent and we can no longer afford the comfortable but extravagantly wasteful older hit-or-miss methods either in training that talent or in pursuing research itself.

The outstanding chapters of this book are, I believe, undoubtedly Chapter 4, "The Design of Experiments" and Chapters 8, 9, and 10 on "Analysis of Experimental Data," "Errors of Measurement," and "Probability, Randomness and Logic." In these chapters, by easy stages, the author introduces the scientist to the best and most useful new ideas that the mathematical statisticians have been developing, ideas indeed of great utility and pertinence to the research man. It is a rather disturbing fact that to date these ideas and the experts who developed them have actually been appreciated and put to work far more widely by business and industry in such fields as quality control of a mass-produced item than by the scientist who could profit so well by them to systematize and minimize his research effort. If you have never heard of the factorial design of experiments, or of significance levels, or of confidence intervals, or of randomization, or the uses of latin squares, or of fractional replication, or placebos, or of the "power" of a test, or of errors of the first and second kinds, or of sequential analysis, perhaps you have been missing something and had better read this book. I wonder, for instance, how many physicists have worried really seriously about the question as to the number of replications that are desirable or necessary in their measurements.

The famous Michelson, Pease, and Pearson experiment measuring the speed of light in a mile-long vacuum tube on the Ervine Ranch in Southern California is a case in point. Some 2885 replications of the time of transit were made with a rotating mirror over a period of three years. The base length was also repeatedly measured, *but far less frequently.* Among the *mirror* measurements, deviations from the mean as large as 30 km/sec were not uncommon and the average deviation was 10 km/sec. Because of the great number of replications a much smaller probable error (±4 km/sec) was assigned to the grand mean, $c = 299774$ km/sec, however. There is now convincing evidence from subsequent work by far superior methods that this mean value was 18 km/sec too low—a systematic error which 2885 replications did not suffice to obliterate! The distribution of the deviations was symmetric about the mean value but decidedly not Gaussian having much too widely extended wings with a sharp central core as though the measurements were taken under two distinctly different conditions, "good" and "bad." Also disturbingly large fluctuations in base length were observed which exhibited some correlation with the ocean tides.

Hindsight is much better and easier than foresight, of course, but one cannot but wonder whether the services of an astute mathematical statistician armed with some of the modern methods of sequential analysis and factorial design might not have saved a great deal of useless replication of the observations in this famous experiment which may now be said to have misled the world about its most vital fundamental physical constant for a period of twenty years.

There are gratifyingly few flaws in this text, most of these being merely faults of omission, but one exception to this is the statement in paragraph 9.8 on rejection of observations which I quote from the top of page 257. "Another, and probably better rule is to take the mean of all but the highest and lowest values." Without any specification of the number of replicate observations to which it is to apply this is extremely dangerous advice to give to neophytes in research. It is only one step away from the "fallacy of the best two out of three." Those who do not know of this pitfall are urged to get a copy of the one page note (10 cents), "The Fallacy of the Best Two Out of Three" Reprinted from the July 1949 *Technical News Bulletin* of the National Bureau of Standards. It has been shown both by experiment and theory that in a sample of only three replicate measurements, a situation in which the most outlying value is nineteen times farther from the intermediate one than is the other extreme value, has a statistical expectancy of occurring in about one out of every twelve sets! The temptation to reject even very outlying values is therefore seen to be a most dangerous one if the sample is small. A famous statistician has said (no doubt with a delightful chuckle!), "If you *must* reject data, reject that which lies closest to the mean because it has the least effect on the mean value." It is my long standing experience that young graduate students inevitably want to make their results look more coherent by rejecting outlying observations, and intuition is a very poor guide in this matter. The boundary line between honesty and dishonesty is hard to discern here and it is far safer to be very sparing about rejection. My own conviction is that the only valid reason for rejecting an observation should be knowledge, *completely independent of the magnitude of the deviation from the mean,* that some condition under which the observation was taken honestly invalidates it for use.

The section 9.11 entitled "Compounding of Errors" was distinctly disappointing in that only the formula for the error of a function, $f(x_1, x_2 \ldots x_n)$, of *observationally independent components* x_i is treated. There are many practical instances, however, when the x_i are observationally correlated (for example when the x_i are the output results of a

least-squares adjustment or even more commonly when they simply consist of several numerical values with their associated probable errors or standard deviations *in whose computation certain common error contributing components have played a role*). Instances are legion where a completely incorrect error measure is assigned to a calculated function of such correlated variables because of failure to take account of the cross-product terms in the generalized propagation of error formula. One way of avoiding this pitfall is to take the trouble always to re-express the function f in terms of observationally independent quantities and to compute the standard error in f from the standard errors of these. Even this elementary precaution is not emphasized however in Dr. Wilson's text.

My experience has been that not one in ten graduate students in physics, even those who have had courses in mathematical statistics and adjustment of observations, knows how to compute the standard deviation of a weighted mean value (1) by internal consistency (based on the error measures of the individual values), (2) by external consistency (based on the deviations of the several values from their mean), and their ignorance as to the distinction between these two error measures or the significance of their ratio is equally abysmal. Another extremely common mistake is to confuse the standard deviation of a single observation with the standard deviation of the mean. Simple every-day homely considerations such as these are insufficiently emphasized in most courses and textbooks and it is a little disappointing to find that Dr. Wilson's otherwise excellent new book suffers from this same defect. The text completely omits any account of how to analyze the residues of the observational equations in a least-squares adjustment, for example, or how to compute the error measures, by external or internal consistency, of the least-squares adjusted output values. Indeed perhaps the most severe criticism that can be made of Dr. Wilson's book is that while the upper crust is well done the under crust of simple elementary principles and practices, which every college senior in the sciences ought to know but which for some reason 95 percent have not absorbed, are omitted or insufficiently emphasized. Dr. Wilson sins in very good company in this respect, however, and the prospective purchaser should not be discouraged by these remarks from making this very worth-while addition to his library.

JESSE W. M. DUMOND
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Heat Transfer Phenomena. R. C. L. BOSWORTH. Pp. 211 +xii, Figs. 41, Associated General Publications, Sydney, Australia (obtainable through John Wiley and Sons, Inc., New York) 1952. Price $6.00.

This volume constitutes a summary of the methods of heat transfer. Although it is too brief to be considered a treatise on the subject, the references to more detailed discussions both in physics and in engineering are so complete that the book serves a definitely useful purpose as a starting point for anyone who wishes to become familiar with some or all phases of the subject. Detailed mathematical developments are kept to a minimum and only methods of attack on a problem are given.

The descriptions are clear without the verbiage which too often makes for cumbersome reading.

In the introductory statement of Chapter 1 the author offers an explanation as to why so few physicists have bothered themselves with problems of heat transfer in their "search of an easy path to academic distinction." It is the reviewer's observation that such a path soon becomes jammed by traffic so that it no longer is easy. In Chapter 8 a discussion is given of the equivalent electrical circuit for a transport set-up, and in Chapter 9 a brief but excellent discussion of the thermodynamical similarity is given.

H. B. WAHLIN
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Essentials of Microwaves. ROBERT B. MUCHMORE. Pp. 236+vi, Figs. 202, 18×23.5 cm. John Wiley and Sons, Inc., New York, 1952. Price $4.50.

Hand-size microwaves with interference patterns that can be measured on an ordinary ruler will in the next generation provide a concrete picture for the understanding of the propagation of light waves and radio waves.

Microwaves are the portion of the spectrum studied most recently. The physicist of this generation approaching microwaves from the field of physical optics looks for and finds the optical diffraction patterns, which, instead of being microscopic, are spread out over the table top. The transmission-line engineer brings the impedance concept into the field of microwaves. Transmission lines with their quarter- and half-wave sections that were spread over the countryside are now set up on the table top. The physicist and the transmission-line engineer hold the two handles of a nut cracker to crack microwave problems. To work together effectively they must understand each other's points of view.

Robert B. Muchmore's *Essentials of Microwaves* is designed to explain qualitatively and diagrammatically the behavior of microwaves and to present microwave techniques to the engineer who is approaching from the point of view of transmission lines and radiofrequency. It also serves to present the engineer's point of view to the physicist.

The book may be divided into three parts and read in three sittings. The first part lays a background of electricity and magnetism and electromagnetic waves in wave guides, a treatment similar to that in Simon Ramo's excellent little book, *Introduction to Microwaves*. The second part deals with the current development of tubes particularly as designed for use in the first low level stage of high gain amplifiers. The third part deals with applications of microwaves to relays, radar, electron accelerators for nuclear studies and microwave spectroscopy as a means of studying molecular structure.

The liveliest part of the book is the second or middle section, which portrays the current research on high frequency electronic tubes. In the field of physics are the

problems of interaction of waves and electron streams. Even the behavior of the simple diode has not been explained quantitatively at high frequencies. In the field of engineering development that is shown the race to determine which tube will serve best as a first stage amplifier in coming relay systems. The important properties of the tube in this race are low noise, power gain, and maximum width of band in the spectrum.

Natural static is missing at microwave frequencies so that tube noise becomes the limitation to high gain amplification. The race between low stage amplifiers is shown graphically on a plot of noise figure against frequency. Currently, crystal mixers used in superhetrodyne reception are in the lead for low noise. The data for this graph were out of date before the ink was dry. However, that does not hurt the graph. It still serves as a score card on which to follow the race. Muchmore presents the case for each tube, the triode, the klystron, the traveling wave tube, the multiple stream tube, and the magnetron, but it appears that he is placing his bets on the traveling wave and multiple stream tubes.

C. L. ANDREWS
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Cosmology—Elements of a Critique of the Sciences and of Cosmology. FERNAND RENOIRTE (translated from the second revised edition by James F. Coffey). Pp. 256+xv. Joseph F. Wagner, Inc., New York City, 1950. Price $3.50.

Scientists have come to realize that the scientific method, indiscriminately applied, may easily lead to results that are incompatible with experience—often even ridiculous. While some of the jibes contained in Standen's *Science is a Sacred Cow* may be clever rather than well-grounded, any thinking scientist who has read the book must agree that all too many of them have a basis in fact. There exists much groundless faith in the infallibility of the conclusions reached by scientists.

But if the scientific method needs to be applied discriminately, what are to be the principles of discrimination? To what type of problem may the so-called "scientific method" be validly applied? Such questions call for a critique of the methods of science and for an investigation of the meaning and scope of the terms used by science. Renoirte's book is an answer to the call although the title, *Cosmology*, would not, of itself, indicate that this is so. The book actually contains little of what is usually accepted as cosmology. Its sub-title, "Elements of a Critique of the Sciences and of Cosmology," more clearly indicates the nature of the work.

Utilizing the operational definition of physical properties, Renoirte outlines a workable meaning for science without violating common sense. Even more important, he realizes why operational definitions are called for in physical science and therefore does not try to apply the conclusions of physical science indescriminately to problems in other fields, the elements of which cannot be adequately described in the terms of physical science.

The book is divided into three parts. The first part (97 pages) contains a critical study of some questions in positive science, not from the point of view of positive instruction, but rather to present detailed examples of the scientific method of developing a physical theory, with an eye to analysis of that development in the second section. A few of the statements concerning atomic theory in this first section are seriously in error, but these do not interfere with its illustrative purpose. The second part (76 pages) is the one which will be of most interest to scientists, and to science teachers in particular. It contains a critique of scientific knowledge proceeding from things and facts through laws to theories. The final section (65 pages) deals with cosmology. Mechanism, dynamism, and hylomorphism are discussed. Although every scientist is something of a philosopher, the material of this section is farther removed from the everyday work of the scientist than that of the preceding section, and is therefore likely to be less fascinating—and more difficult to comprehend.

In the opinion of this reviewer fundamental principles are expressible in fundamental language; Renoirte's *Cosmology* is a proof that this can, at least, be true. Renoirte manages to do an unusually good job of keeping both feet on the ground despite the fact that he has a Sc.D. in the physical sciences and mathematics as well as a doctorate in philosophy.

The material of this book should be familiar to anyone who professes to teach others the place of science in the intellectual world. The relation of physical concepts to objective reality, the meaning to be assigned to physical laws and theories are problems which cannot be ignored by a worth-while teacher of physics. What answers shall be given to such common student questions as "Do electrons *really* travel in elliptical orbits?"

Here is a book which does not pretend to solve the problems of physical science. Its purpose is rather to bring out more clearly the nature of these problems, thereby preparing them for solution. In this it is largely successful.

R. BARTHEL
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Nuclear Stability Rules. N. FEATHER. Pp. 159, Figs. 27, 14×22 cm. Cambridge University Press, New York, 1952. Price $4.00.

The aim of this monograph as stated by the author in the preface has been "to survey the results of experiments interpreted as providing information concerning the stability properties of the ground states of nuclei with a view to eliciting the significant regularities." To this end the author has organized the known data into three large chapters entitled "The Systematics of Stable Nuclei," "Regularities in a Disintegration," and "Regularities in β Disintegration." A small (four-page) chapter is devoted to spontaneous fission.

In Chapter 1 after a thorough analysis of the data concerning the neutron, proton, and mass numbers of stable nuclides, the author presents the rules which have been inferred from the breakdown of stable nuclides into classes

specified by these numbers. Some of these rules are well known; however, many are either new or are expressed more precisely than they have been previously.

Chapter 2 is devoted to discussions of the relationship between lifetime and alpha-disintegration energy, the systematics of beta-stable alpha-emitters and to the energetics of alpha-decay particularly in relation to shell discontinuities. Extensive use is made of Geiger-Nuttall diagrams to reveal the general regularities in alpha-decay which are so well explained by the Gamow, Gurney, and Condon theories. These same diagrams also reveal significant departures from theory which even today defy quantitative explanation.

In Chapter 3, similiar use is made of the Sargent lines as a means of testing beta-decay theory in the light of data concerning lifetimes and beta-energies. The energetics of beta-decay and its bearing upon odd-even regularities in nuclei and shell-structure effects also come in for discussion in these chapters.

In this work Professor Feather has attempted to bridge the gap which exists between the nuclear experimentalists who are accumulating data with great rapidity and the theorists who are already overwhelmed by the existing accumulation. Expressed in his own words, Dr. Feather's aim "has been so to marshall the experimental facts that the theorist is most likely to be inspired by valid—even correct—ideas on contemplating them." This seems to be an excellent definition of a type of endeavor which has been so important to the development of physics that it would seem worthy of a special name, distinct from either experimental physics or theoretical physics. It would seem that the name phenomenological physics would be appropriate as the general classification for this type of research activity. As an illustration we cite the example of the great "phenomenologist," Kepler who organized the observations of the experimentalist Tycho Brahe into simple laws which were later used by the theorist Newton as an aid in arriving at and verifying his theory of universal gravity. Numerous other illustrations may be cited which indicate the importance of this middle man or this middle step to the advancement of physical understanding. Although often the three steps are mutually dependent and often a single scientist takes two or even three of the steps, nevertheless, the types of physical and intellectual activity involved would appear to be well enough defined in each case to warrant the three classifications.

Of course, time alone will tell whether Dr. Feather's "marshalling" of the experimental facts will bear fruit. The reviewer believes that many of the rules and regularities concerning the energetics or transformations involving ground states of nuclei are most clearly revealed by relating them to the atomic mass surface, whereas Dr. Feather refers only briefly to this concept. The monograph contains no references to the remarkably successful spin orbital coupling model of Mayer and Haxel, Jensen and Suess. This perhaps was in conformity with Dr. Feather's expressed intention to avoid any attempt to interpret the facts concerning magic numbers in terms of a shell model. It would seem, however, that the time is ripe for precisely such an attempt.

Taking the monograph as a whole, the reviewer has found Dr. Feather's unique collection and organization of the data and his concise and lucid comments on the data to be valuable contributions to the literature of nuclear physics.

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The Elements of Nuclear Reactor Theory. SAMUEL GLASSTONE AND MILTON C. EDLUND. Pp. 416+vii, Figs. 87, 15.5×23.5 cm. D. Van Nostrand Company, Inc., New York, 1952. Price $4.80.

An urgent need for more complete coverage of the theory of nuclear reactors has arisen since 1950. Several factors combined to create this situation:

(a) An increased emphasis on reactor development within the U. S. Atomic Energy Commission program.

(b) The intermittent release of previously restricted constants and calculation methods.

(c) The interest in reactors of industrial concerns for power applications and of educational institutions for research purposes.

The Elements of Nuclear Reactor Theory, by Glasstone and Edlund, provides an excellent step forward toward the goal of making available all unclassified information on reactor theory and design. Earlier treatments of reactor theory available to the public described adequately the theoretical information which could be released at the time they were written. As soon, however, as the scope of information was enlarged, and the interest in reactors outside the U. S. Atomic Energy Commission gathered momentum, two uses developed which could not be served by the few existing treatises:

(1) The need on the part of reactor design groups for a conveniently arranged reference book which supplied working formulas for use as a starting point for design calculations, and (2) the need on the part of training groups for a coherent, unified treatment of reactor theory which could be used as a textbook. In addition, there was needed within the atomic energy program itself a single compilation of fundamental theory of nuclear reactors that was well organized and employed consistent notation. *The Elements of Nuclear Reactor Theory* will serve to meet all these needs.

The outstanding feature of this book is the clarity, orderliness, and completeness with which the material is presented. This is true of the over-all plan of the book and in the treatment of each separate topic. The origin and method of the mathematical formulation is explained, and the subsequent mathematical manipulation is presented in satisfactory completeness. Each paragraph and many equations are numbered in an easily followed system to facilitate use of the book as a reference text. The authors of this book are to be highly complimented on the simplicity of style and the clarity of presentation which they have achieved.

The first three chapters are devoted to nuclear structure and stability, nuclear reactions, and production and reactions of neutrons. This material is intended to constitute a rapid review of general nuclear physics. One would hardly undertake a serious study of reactor theory, however, without a considerably more general acquaintance with nuclear physics than is presented here. Thus this subject matter appears not to be needed, although it in no way detracts from the book.

The special properties of neutrons described in the last part of Chapter III are, of course, directly applicable to the subsequent theoretical analysis. Chapter IV is a concise, readable description of the mechanism of fission and its consequences. Chapters V and VI on the slowing and diffusion of neutrons are preparatory to the theories of the homogeneous systems, unreflected and reflected, developed thoroughly and systematically in Chapters VII and VIII. Nothing is lacking in the theoretical presentation. The value of these sections would be greatly enhanced, however, both for convenient use as a textbook and as a reference handbook if a few numerical examples and illustrations were included. For example, the calculation of group constants for a reactor of particular composition and a sample calculation of critical dimensions and critical mass by the age-diffusion method would be very helpful. This, of course, is not essential to an understanding of the theoretical principles involved.

These same comments may apply to Chapter IX, "Heterogeneous (Natural Uranium) Reactors." It is the experience of these reviewers that in an actual situation the student or the design engineer is faced with a difficult task in deciding how to choose and use appropriate values in the calculations.

Chapters X and XI contain, respectively, a quite satisfactory treatment of time behavior of a bare thermal reactor and control rod, temperature and fission product factors in reactor control.

The last three chapters, XII, "General Theory of Multiplying System;" XIII, "Perturbation Theory;" and XIV, "Transport Theory and Neutron Diffusion," deal with advanced methods in nuclear reactor theory. The authors have made a good selection of portions of the theory that are not too complex for the graduate physicist or engineer to follow. The notation is straightforward, and enough of mathematical operations are supplied for good continuity.

In the suggestion above that inclusion of additional numerical illustrations and practical appliactions would have enhanced the value of this book, it was not intended to criticize the authors for omission of material not necessarily relevant to the purpose in mind when the book was written. The objective of the authors of presenting an introduction to the essentials of nuclear reactor theory with coherence and clarity has been admirably achieved. Student engineers and reactor designers will accept this work with gratitude, and be content to wait until someone else takes up the task of filling in the substantial gap which still remains between theoretical principles and practical applications.

CLIFFORD BECK AND RAYMOND L. MURRAY
North Carolina State College

Mesons—A Summary of Experimental Facts. ALAN M. THORNDYKE. Pp. 237+viii, Figs. 106, 16×23.5 cm. McGraw-Hill Book Co., Inc., New York, 1952. Price $5.50.

The careful reader of this book will indeed find a reasonably complete summary of experimental information available on mesons. It is complete in the sense that most of the relevant work is mentioned and referred to and some of the more easily digested results are briefly quoted. It does not attempt a detailed discussion of very many items in the imposing list of references.

The diversity of the experimental material on mesons demands some guiding principle in the organization of the book. The author has chosen an essentially logical or didactic type of organization. Thus Chapter 1 provides the original evidence for the existence of mesons in cosmic rays and Yukawa's ideas on nuclear forces. Chapter 2 reviews the data on the properties of the cosmic-ray (μ) mesons such as mass, charge, and spin. Chapter 3 discusses new types of mesons, π and others. The discussion of the heavier particles is of course already out of date. As far as the π-meson is concerned, only a small part of the findings (mass by grain counting) of the basic experiment in its discovery are given in Chapter 3 since the other findings can be logically included in later chapters. Chapter 4 discusses some of the fundamental experiments on artificial production of mesons, gives the mass of the π-meson, and describes the discovery of the π^0 meson. In Chapter 5 are collected data and experiments on meson decay. Here are the μ-lifetime, its change for negative charge and larger Z, and its decay scheme. The decay experiments on the π-meson include the range of μ-mesons from π-decay and the mass of the π, the decay scheme, and lifetime, and the decay of π^0 mesons. Chapter 6 covers the interaction of mesons with matter, both electromagnetic and nuclear. The Z^4 law of capture of μ-mesons is here brought out for the first time. Experiments on the star production by π-mesons and the capture of π-mesons in hydrogen are discussed. The remaining chapters, 7 and 8, deal with the production of and experiments on mesons in cosmic rays. The subject matter here is of great diversity and is still only poorly digested.

The arrangement of the book, as outlined above, permits fairly ready access to the material as a reference source, and it is probable that the book will find its greatest use as a reference on meson experiments and as a guide to the literature.

The organization of the material has, however, also brought with it some very serious disadvantages. The fact that the original and striking experiments on π-μ decay, for example, are dissected into three parts in as many chapters, prevent any appreciation or understanding of the work and leaves only a dry collection of apparently unrelated facts devoid of interest to the reader. Nuclear capture of μ-mesons has received similar treatment with the same result: what was an exciting and important discovery is presented in such a way as to not only be without interest but also so that an understanding of the interrelation of various types of experimental results is not apt to be noticed.

The book also suffers from the almost total lack of theoretical discussion and argument. Even although no complete or adequate theory of mesons exists there are many points—such as meson spin and the selection rules in meson capture in hydrogen—where a little theory is very illuminating and can very materially help to tie together what is otherwise, as it is in this book, an undigested collection of facts.

R. F. CHRISTY
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Calculus—A Modern Approach. KARL MENGER. 255+xxv, Figs. 46, 22.5×29 cm. The Bookstore, Illinois Institute of Technology, Chicago. Price $4.50.

The *Calculus* of Karl Menger covers about the same material as the usual text used in American universities for the elementary calculus courses. In treatment of material and in matters of notation, it is as different from the usual text as day is from night.

In the development of mathematics, notation has undoubtedly played a role whose importance is often underestimated by the working mathematician. Failure to use the appropriate notation for a mathematical concept. has often hindered progress to a great extent. Any book on the history of mathematics is full of examples of this sort. After perusing Menger's book, one is almost inclined to wonder how calculus could have developed at all since the days of Newton and Leibniz.

In several papers and books, published during the past decade, Menger undertook what he calls a semantic clarification of the notation of the calculus. In the Preface to his *Calculus*, he states the basic ideas and principles which guided him in this work. Rather than restate these principles here, it seems more appropriate to give a few examples.

The usual classical notation for a function, $f(x)$, is replaced by writing simply f, thus freeing this symbol from the number x. This has been done frequently in mathematical literature, but in such cases it merely served as an abbreviation for the "full" notation, $f(x)$. Menger uses even the notation sin, cos, log, etc., without the x, thus emphasizing that each of these "pure" functions, as he calls them, represents a symbol for the association of one real number y to each value of x in a certain interval. For one of the most important functions, denoted by $y=x$ in the classical notation, Menger had to introduce the special symbol I (identity). Thus, what used to be the function $y=2x^2-x-3$ becomes $2I^2-I-3$. For the exponential function $y=a^x$, Menger uses the notation $\exp_a$. The value which the function assumes for the number x is denoted by fx, just as we write today $\sin x$ or $\log x$. Menger distinguishes between $f\cdot g$ and fg; the former means the ordinary product of the functions, the latter is the result of substituting the function g into f, $f(g(x))$ in the classical notation. For the derivative, Menger allows only one notation, $\mathfrak{D}f$, already known today from its use in most texts on differential equations. A particularly praiseworthy improvement is the distinction between $\mathfrak{D}^{-1}f$, the antiderivative (indefinite integral) of f, and

$$\int_a^b f,$$

the integral from a to b of f (definite integral). Note that $(x)dx$ is omitted from the integral as being entirely superfluous. Numerous other examples might be given. But it seems clear from the examples above that each change of notation is based on a desire to clarify the mathematical concepts involved.

One item of particular interest to physicists is the distinction made by Menger between "pure" functions and "descriptive" functions. Any relation between two geometrical or two physical quantities, such as the radius and volume of a sphere, or the pressure and temperature of a gas, is considered a descriptive function, connecting two measurable quantities with one another. It is for such quantities that the word "variable" is reserved. Unlike the case of pure functions, such as I, sin, $\exp_2$, it is necessary in the case of a descriptive function to indicate both variables; thus $p(\theta)$ may mean the pressure p as a function of the temperature θ, $p(t)$ the pressure p as a function of the time t.

The treatment of the calculus is also quite different from that in the usual textbook. The reader is at once introduced to the idea of the inverse roles of the slope of a curve and the area under the curve in the case of linear functions (straight lines). The idea of the limit is introduced in a manner which is at once simple, clear, and appealing to a beginner. Later the treatment of the calculus of pure functions practically begins with what is now known as the fundamental theorem of the calculus, with which the reader is already familiar in the case of linear functions. An entire chapter is dedicated to the application of calculus to physical sciences. Other subjects covered in the book include the usual rules of differentiation, technique for finding antideratives, mean-value theorem and Taylor's expansion. The book ends with a chapter on partial differentiation.

The book contains a large number of exercises for the student; it is written in very clear language and has many examples which serve to make the basic concepts clear to the reader.

It is, of course, inevitable that a new notation, such as that used by Menger, must have its shortcomings compared to the classical one to which we are used today. For instance, it is undeniable that the substitution rule (rule for the derivative of a function of a function) somehow, perhaps purely psychologically, appeals much more in the old, almost visible form

$$\frac{dz}{dx}=\frac{dz}{dy}\cdot\frac{dy}{dx}$$

than in the new form $\mathfrak{D}(fg)=\mathfrak{D}fg\cdot\mathfrak{D}g$; this at first is mysterious even to the experienced mathematician, until he realizes that $\mathfrak{D}fg$ means $(\mathfrak{D}f)g$ and is the result of substituting g into the derivative of f.

Will Menger's new notation find its way into the mathematical (and other scientific) literature? The difficulties, mentioned by Menger, himself, in the Preface, are undoubtedly great, particularly if one considers the fact that for two centuries the present notation has been used

and that the entire literature in mathematics and its application is written in it. Nevertheless, the book contributes a great deal to the clarification of the entire concept of the Calculus, and it is to be hoped that it will soon appear in printed form, so that many professional mathematicians can profit from reading it and then begin to spread the new gospel to physicists and other scientists, and finally to the students.

FRITZ HERZOG
Michigan State College

Mechanics, Lectures on Theoretical Physics, Vol. I. ARNOLD SOMMERFELD, translated by Martin O. Stern. Pp. 289+xiv, Figs. 58, 23.5×16 cm. Academic Press, Inc., New York, 1952. Price $6.50.

Among those active in the early development of quantum mechanics Professor A. Sommerfeld was both a leader and a teacher. Soon after Bohr gave his first analysis of the hydrogen spectrum, Sommerfeld took up the matter of atomic spectra, and gave his famous treatment of the hydrogen fine structure. He followed this with analyses of the Zeeman effect and the general problems of series spectra and multiplet systems.

Above all he wrote his book, *Atomic Structure and Spectral Lines*, which introduced a whole generation of young physicists to the older quantum theory and to the problems of theoretical spectroscopy. Probably more than any other publication, this book was instrumental in setting the stage for the new quantum mechanics of 1925. The difficulties of the embyro theory were not concealed, but the emphasis was on the successes in such a way as to convince the student that further progress was not only possible, but was imminent. Not only through his book but personally, Sommerfeld was the teacher of a whole school of theoretical physicists. His influence still survives vigorously in many of his former students.

Knowledge of the quantum theory was spread widely and effectively by Sommerfeld, but his own standing and reputation were not built on it alone. His reputation as a mathematical physicist had been established much earlier on the basis of his four-volume work with Klein on the spinning top, extensive work on the propagation of electromagnetic waves, and contributions to other classical subjects.

As Sommerfeld's research was founded on a solid base of classical mathematical physics, so his teaching included a sequence of courses of lectures on the classical subjects, each accompanied by its "praktikum" or theoretical laboratory. During the last years of his life these lectures were written up and the first series on *Mechanics* is now available in English.

The book shows the characteristics that so distinguished the author. The mathematics is elegant, but the physics is kept in the forefront and made obvious by numerous very practical illustrations. Rather unusual in such a book from Europe is the set of problems, further amplifying and illuminating the text. The use of these constituted an integral part of the author's teaching.

This book is not a beginner's introduction to mechanics, although it may be regarded as an introductory treatment of advanced mechanics. Full attention is paid to the general principles and the formulation of mechanics, including Hamilton's principle and the Hamiltonian equations. The principles of d'Alembert and of virtual work are given central positions, and most problems are formulated in terms of one or the other. Sommerfeld's meticulous attention to the basic ideas is shown in his discussions of these principles. The generality of the formulation is never allowed, however, to obscure the fact that mechanics is intended to be applied to specific problems.

A word of commendation is due the translator who has rendered Sommerfeld's elegant German into pleasantly readable English. None of the effectiveness is lost in this process.

The book is intended to be a text, not an exhaustive treatise, but it provides a very satisfying basis for one who wishes to apply mechanics to engineering problems or to problems of atomic physics.

W. V. HOUSTON
The Rice Institute

New Members of the Association

The following persons have been made members or junior members (*J*) of the American Association of Physics Teachers since the publication of the preceding list [Am. J. Phys. 21, 238 (1953)].

Adams, Gerald Edwin, R. F. D. 1, Valparaiso, Ind.
Andrisek, John Richard (*J*), 144 Marlborough St., Portland, Conn.
Bentley, Bedford Tate (*J*), 1912 McKean Ave., Baltimore, Md.
Blattner, Donald James, 435 W. 119th St., New York 27, N. Y.
Bloch, Ingram, Box 48, Vanderbilt University, Nashville, Tenn.
Bracewell, Kent H., Hamline University, St. Paul 4, Minn.
Chavis, Alex, Box 527, Jefferson City, Tenn.
Cummins, Richard Lee (*J*), 504 South Busey, Urbana, Ill.
Domitilla, Sister Mary, 815 S. Ninth St., LaCrosse, Wis.
Early, James M., 230 W. 4th St., Emporium, Pa.
Edwards, Steve, Jr. (*J*), 112 S. Woodward St., Tallahassee, Fla.
Fletcher, Charles H., 101 Transportation Bldg., University of Illinois, Urbana, Ill.
Galphin, Bruce M. (*J*), Box 392, Florida State University, Tallahassee, Fla.
Garfunkel, James H., P. O. Box 431, Boulder, Mont.
Gordon, Clifford Mark (*J*), Scott Laboratory, Wesleyan University, Middletown, Conn.
Greene, Lorenzo Philip (*J*), 311 Elm St., N.W., Washington, D. C.
Grim, Samuel Oliver, Lebanon Valley College, Annville, Pa.
Gruenzweig, Simon, 817 W. 13th St., Little Rock, Ark.
Gunton, Robert Carl, Department of Physics, West Virginia University, Morgantown, W. Va.
Hadley, Lawrence N., Jr., 22 Occom Ridge, Hanover, N. H.
Howard, William Hamilton (*J*), Scott Laboratory, Wesleyan University, Middletown, Conn.

Hutchenson, Dwight Ralph (*J*), Box 901, Clemson College, Clemson, S. C.
Jongedyk, Richard Allan (*J*), 33 E. Healey, Champaign, Ill.
Kerman, Ralph Owen, Kalamazoo College, Kalamazoo, Mich.
Kortright, James McDougall, Department of Physics, Purdue University, Lafayette, Ind.
Ledbetter, Nasira F. (*J*), Wheatley Hall, Box 427, Howard University, Washington 1, D. C.
Lee, Charles J., 2456 N. 16th St., Philadelphia 32, Pa.
Love, G. Ross, 52 Bullock Ave., Ottawa 1, Ontario, Canada
McConnell, Robert A., Jr. (*J*), 201 N. Market St., New Wilmington, Pa.
McManus, George Marty (*J*), 14083 Glenwood, Detroit, Mich.
Muller, Burton Harlow (*J*), Department of Physics, University of Illinois, Urbana, Ill.
Neufeld, Murray J. (*J*), 1277 Washington Ave., Bronx 56, N. Y.
Rodman, James Purcell, 1192 Sunset Dr., Alliance, Ohio
Rosengren, Jack Whitehead, Department of Physics, Massachusetts Institute of Technology, Cambridge 39, Mass.
Ryan, Rev. Kevin, O.S.B., St. Meinrad, Ind.
Schearer, Laird D. (*J*), 2116 Allen St., Allentown, Pa.
Skeel, William Lewis (*J*), 3 Scott Laboratory, Middletown, Conn.
Snyder, Edward Charles, 522 Monte Vista, Azusa, Calif.
Verderber, Rudolph Richard (*J*), 731 W. Lafayette St., Tallahassee, Fla.
Walker, Edward Corbell (*J*), Department of Physics, Howard University, Washington, D. C.
Williamson, Mabel Paige (*J*), 1927 Second St., N.W., Washington, D. C.
Wilson, Leland Leslie, Box 295, Collegeboro, Ga.

AMERICAN JOURNAL OF PHYSICS VOLUME 21, NUMBER 5 MAY, 1953

RECENT MEETINGS

Chicago Section

The autumn meeting of the Chicago Section of the American Association of Physics Teachers took place on December 6, 1952 at the Lawson YMCA, Chicago, Illinois. There were twenty-three members and guests present. After the luncheon a program entitled **Physics Around the World** was presented, which consisted of the following three invited papers:

Physics in Burma. THAN TIN, *University of Rangoon, Burma.*

Education for physics in Spain. SALVADOR FERIGLE, *Illinois Institute of Technology.*

My experiences teaching physics in Indonesia. SYBRAND BROERSMA, *Northwestern University.*

A short business meeting followed an extended discussion of the papers.

The following officers were elected:

President, R. L. PRICE, *Joliet Junior College*, Vice-President H. C. JENSEN, *North Park College*, Secretary, LOWELL C. WARNER, *Wilson Junior College.*

THEODORE G. PHILLIPS
Secretary

Western Pennsylvania Section

Meeting on December 13, 1952 at the Buhl Planetarium, Pittsburgh, Pennsylvania, were 128 guests and members of the Western Pennsylvania Section of the American Association of Physics Teachers. The main feature was **Try It and Think** by guest speaker, DR. RICHARD M. SUTTON, *Haverford College*. The series of demonstrations, in the masterful Sutton manner, was greatly enjoyed by the group.

Contributed Papers and Other Program Items

Welcome. ARTHUR L. DRAPER, *Director of the Buhl Planetarium.*

A dynamic coefficient of friction experiment. ROBERT RESNICK, *University of Pittsburgh.*—In the usual experiment to determine the coefficient of sliding friction one measures directly the force required to keep a block moving at constant speed over a horizontal surface. Aside from difficulties in achieving this condition under usual laboratory conditions, such a "static" experiment teaches the student very little fundamental physics. A "dynamic" experiment is described in which an accelerating mass M falls through a distance s as it pulls a mass m along a horizontal surface. The kinetic energy acquired by m is dissipated by friction in bringing m to rest after it slides a distance d from the position at which M ceases to act. One obtains for μ, the coefficient of sliding friction,

$$\mu=\frac{s}{(M+m)d/M+(m/M)s}.$$

One can choose M so as to use the maximum available distance d, thereby minimizing errors due to length measurement and lack of surface uniformity. This dynamic method is simpler to perform, has smaller experimental uncertainties, and illustrates more fundamental mechanical principles than the static method. Results for a wide

range in d show that the dependence of μ on velocity introduces no more variation in the measured μ then is obtained in the static experiment.

A demonstration of Ohm's law and simple dc circuits. ROBERT M. WOODS, *Westminster College*.—Several years' teaching experience has shown that the average student in beginning physics courses seems to have difficulty in understanding Ohm's law as it applies to simple direct current circuits. A demonstration board has been devised on which simple circuits may be set up, and by use of lecture-type ammeters and voltmeters, the current and potential relations may be shown. The author has used a board of this general type for several years with considerable success. The present form is the result of a paper given at the New York meeting of the AAPT last January.[1] Special snap connectors[2] are used to facilitate making and changing connections. The ammeter may be connected at any point in the circuit to show that in a simple series circuit the current is not "used up in passing through a resistor." The voltmeter can be connected between any two points to show that the sum of the potential drops is equal to the applied potential. Parallel circuits can be set up and the same laws shown. It has been found that the snap connectors have a very small resistance and as a result the computations are limited only by the accuracy of the meters used.

The use of the same snap connectors in the electronics laboratory is shown. This leads to ease in making and changing connections and eliminates the danger which results from loose connections.

[1] T. J. Blisard and B. A. Greenbaum, Am. J. Phys. **20**, 399 (1952).
[2] "Nu-Snap" Connectors, A. L. Seinsoth Company, 43–47 W. 24th Street, New York 10, New York.

A projection oscilloscope. D. BURK AND T. FIELDS, *Carnegie Institute of Technology*.—This oscilloscope was designed and built for lecture demonstrations at the Carnegie Institute of Technology. It was built around a magnetically deflected 3-inch cathode-ray tube, Schmidt optical system and a 25-kv accelerating supply taken from a Norelco projection TV unit. The amplifiers are good to 10 kc. At a projection distance of 8 ft the picture is 5 ft in diameter with a very brilliant ¼-inch trace. It will demonstrate the following: variable frequency sine wave, Lissajous figures, wave forms from a microphone, hysteresis loop, square and triangular waves.

Star of Bethlehem. *Buhl Planetarium Sky Theater*. The mysterious legends which surround the Christmas Star have puzzled astronomers throughout the ages. No one knows for certain just what heavenly body appeared so majestically above Bethlehem. Was it a comet, a nova, or a meteor? Or was it truly a miraculous star—a sign to men of good will? Blended with Christmas music this reverent and inspiring drama portrays the coming of the Prince of Peace.

Zeiss planetarium projector. ARTHUR L. DRAPER, *Buhl Planetarium*.—An additional special showing of the scientific features of the instrument, for the AAPT members and guests.

The business meeting, after luncheon at Boggs and Buhl Dining Room, was presided over by President WILLIAM C. KELLY. Minutes of the previous meeting were approved as read. Professor CHAS. WILLIAMSON, *Carnegie Institute of Technology*, was elected National Representative to the AAPT. Next year's officers are: President, RICHARD C. HITCHCOCK, *Buhl Planetarium*; Vice-President, ROBERT M. WOODS, *Westminster College*; Secretary, R. L. BROWN, *Allegheny College*.

The invitation of Dr. R. C. Colwell to hold the Spring, 1953 meeting at West Virginia University, Morgantown, West Virginia, was accepted. The invitation of Professor A. J. Kozora to hold the Fall, 1953 meeting at Duquesne University, Pittsburgh, Pennsylvania, was also accepted. Father Bernard Brinker, retiring National Representative to the AAPT, presented a report on the national activities of the association.

A proposal that the Section participate in the reorganized Pennsylvania Conference of Physics Teachers was presented by Dr. Wayne Webb and Dr. Marsh White of Pennsylvania State College. After discussion, Dr. O. H. Blackwood moved that the Western Pennsylvania Section go along with the proposal. Motion seconded and carried.

RICHARD C. HITCHCOCK, *Secretary*

AMERICAN JOURNAL OF PHYSICS VOLUME 21, NUMBER 5 MAY, 1953

Proceedings of the American Association of Physics Teachers

Twenty-second Annual Meeting, January 22–24, 1953

THE twenty-second annual meeting of the American Association of Physics Teachers was held in Cambridge, Massachusetts, January 22, 23, and 24, 1953, with Harvard University acting as host, both to our Association and to the American Physical Society. This represents a departure from a custom of several years standing during which period these winter meetings have been held at Columbia University. Members of the Association seem to appreciate the opportunities afforded by such a joint meeting. Not only is it possible to attend

Physical Society sessions of interest but one is able to renew acquaintances with old friends who are in research.

The facilities of Harvard University did not appear to be overtaxed even though there were as many as 7 simultaneous sessions of the Physical Society. The meetings of our Association were held in Jefferson 250 and in Burr Hall B, both large auditoriums with excellent physics facilities. Burr Hall is a new building of most modern design built for introductory instruction in the sciences for the General Education Program at Harvard. Professor Gerald Holton described its unique features in an invited paper which opened our program Thursday morning. The spacious foyer of Burr Hall served double duty as convention headquarters and as a focal point for meeting one's friends.

The AAPT program consisted of 36 contributed papers (only 2 or 3 of the authors were unable to attend), 2 invited papers, and 2 round tables in addition to the joint ceremonial session and joint banquet with the Physical Society on the Friday. Abstracts of the contributed papers appear in the program printed below. It is hoped that the other papers and addresses will be available shortly for publication in the *Journal*. Besides describing the facilities of Burr Hall, Professor Holton and his assistants presented a number of demonstrations to illustrate the type of demonstration equipment being assembled for the General Education Program at Harvard. In his invited paper, "Some Thoughts on the Educational Process," Dean Elmer Hutchisson of Case Institute of Technology pointed up a number of significant features of the educational process most effectively. His unique graphs are surely not to be found in conventional education literature!

The round table "Physics beyond General Physics for Nonphysics Majors" was presided over by Professor Joseph H. Keenan of Massachusetts Institute of Technology, with a panel consisting of Professor E. G. Rochow of the Harvard Chemistry Department, and Professors R. B. Adler, H. C. Weber, and J. A. Hrones of the departments of Electrical, Chemical, and Mechanical Engineering at MIT, respectively. The discussion was largely concerned with physics requirements for engineering students. The speakers were essentially unanimous in denouncing the plans which some institutions are considering of transferring instruction in physics from the Department of Physics to the School of Engineering. An effort to get the speakers to commit themselves with respect to certain units, notably the slug, was unsuccessful! This led to a request that the general topic of units be made the subject of a session at the next annual meeting.

The "Undergraduate Laboratory Teaching" round table, with Professor S. C. Brown of MIT presiding also elicited considerable discussion. Professor V. E. Eaton of Wesleyan University described his "Aims and Methods in the Introductory Physics Laboratory." Professor H. V. Neher of California Institute of Technology described some "New Sophomore Electricity and Magnetism Experiments." "Laboratory Teaching in General Education Courses" was discussed by Professor C. L. Henshaw of Colgate University, and Professor S. C. Brown described "An Undergraduate Course in Research Techniques."

The Friday afternoon joint ceremonial session was probably the high point of the program. The title of Dean Van Vleck's address, "Two Phenomena of Inversion," given as retiring president of the American Physical Society, had a double meaning. The first portion of his address was concerned with the barrier which current immigration policies of our country have raised to the entry of foreign scientists. The second portion described in chronological order the theoretical developments which have been required to keep pace with the experimental advances in the microwave spectroscopy of ammonia. The Richtmyer Lecture was delivered by Professor Edward M. Purcell of Harvard University on the subject "Nuclear Magnetism." Dr. Purcell was last year added to the list of American recipients of the Nobel Prize in physics.

The Oersted Medal was this year awarded to Professor Richard M. Sutton of Haverford College. In his presentation speech, Professor Mark Zemansky, chairman of the Committee on Awards, reviewed Professor Sutton's considerable contributions to the teaching of physics—notably his success in devising classroom demonstrations. Professor Sutton is a past president of the American Association of Physics Teachers

and a continual contributor to the activities of the Association. The widely used *Demonstration Experiments* which he edited for the Association is still a best seller. Professor Sutton's address "The Heritage of a Physics Teacher" traced his growing interest in the physical world, from earliest boyhood days in Colorado up to the present, with its many added satisfactions in seeing students develop. Most recent Oersted medalists have received the award as they were about to retire. Professor Sutton has many fruitful years ahead.

On recommendation of the Committee on Awards, the Association at its business meeting Saturday morning voted Honorary Membership to two distinguished scientists, R. POHL, Professor of Physics at the *University of Göttingen* and J. H. KEENAN, Professor of Mechanical Engineering at *Massachusetts Institute of Technology*. New officers for 1953 were also announced: DR. P. E. KLOPSTEG of the *National Science Foundation*, president; and PROFESSOR MARSH WHITE of *The Pennsylvania State College*, president-elect. PROFESSORS R. F. PATON and F. W. SEARS continue as secretary and treasurer, respectively. Our 1952 president, PROFESSOR W. S. WEBB, was unable to attend the Cambridge meeting because of the serious illness of Mrs. Webb.

Next summer's meeting of the Association will be held at Pittsburgh, Pennsylvania, June 25, 26, and 27 when the Mellon Institute has kindly put an air-conditioned lecture hall at our disposal. The annual meeting will be returned to Columbia University next January.

R. RONALD PALMER,
1952 Program Chairman

Program

Invited Papers

New lecture room facilities and demonstrations. GERALD J. HOLTON, *Harvard University* (assisted by the teaching staff in introductory physics).

Some thoughts on the educational process. ELMER HUTCHISSON, *Case Institute of Technology*.

Two phenomena of inversion. J. H. VAN VLECK, *Harvard University*. (Address of the retiring president of the American Physical Society.)

The heritage of a physics teacher. RICHARD M. SUTTON, *Haverford College*. (Response of the Oersted medalist.)

Thoughts on technical education stimulated by a visit to Britain. J. A. STRATTON, *Massachusetts Institute of Technology*. (After-dinner speech.)

Richtmyer Memorial Lecture

Nuclear magnetism. EDWARD M. PURCELL, *Harvard University*.

Round Tables

Physics beyond general physics for nonphysics majors (JOSEPH H. KEENAN, *Massachusetts Institute of Technology*, presiding).

E. G. ROCHOW, *Department of Chemistry, Harvard University*.

R. B. ADLER, *Department of Electrical Engineering, Massachusetts Institute of Technology*.

H. C. WEBER, *Department of Chemical Engineering, Massachusetts Institute of Technology*.

J. A. HRONES, *Department of Mechanical Engineering, Massachusetts Institute of Technology*.

Undergraduate laboratory teaching (SANBORN C. BROWN, *Massachusetts Institute of Technology*, presiding.)

Aims and methods in the introductory physics laboratory. V. E. EATON, *Wesleyan University*.

New sophomore electricity and magnetism experiment. H. V. NEHER, *California Institute of Technology*.

Laboratory teaching in general education courses. C. L. HENSHAW, *Colgate University*.

An undergraduate course in research techniques. S. C. BROWN, *Massachusetts Institute of Technology*.

Contributed Papers

1. Potentialities of the introductory physics course in developing mathematical intuition. IRA M. FREEMAN, *Rutgers University*.—The rightful expectation on the part of teachers of elementary physics that students coming to them have a reasonable mathematical background is all too often unfulfilled. This is apt to be the case particularly in terminal college courses for nonscience students. In attempting to make the best of this situation we must not overlook the fact that, reciprocally, the subject matter of general physics offers an unusual opportunity for imparting a feeling for certain fundamental mathematical concepts of wide applicability. A minimum set of such concepts was suggested and briefly discussed.

2. The place of Newton's first law in elementary physics. JAMES L. ANDERSON, *Rutgers University* (Introduced by Ira M. Freeman).

3. Three new formulations of Newton's laws. P. LECORBEILLER, *Harvard University*.—It is possible to formulate four different sets of three laws, two in the field of mechanics and two in the field of electricity, which follow the pattern of Newton's three laws of motion. The first set, relative to a mass particle limited to one degree of freedom, and to an inertial reference point, is that of Newton, in scalar form. The three other sets refer to an electrostatic capacitor, to a spring coil, and to an inductor coil. These four sets of laws are illustrations of the transformation devices known as "analogy" and "duality." They enable us to contemplate Newton's Laws divested of their customary associations.

4. New type quiz questions in physics. JULIUS SUMNER MILLER, *Ford Foundation Fellow, University of California* (By title).—Although the standard types of quizzes and tests—the problem type, the exposition type, the multiple-response, the true-false—have fairly well proved themselves, quizzing and the making out of quizzes gets boring. Students need a change too. The type of quiz here proposed possesses a number of virtues not found in the standard types, particularly in the matter of critical thinking. Moreover, when we go over this kind of quiz, some very excellent teaching is possible, not alone in physics but in logic and analysis.

Type I: Mark the statement *TR* if it is true and if the reason given is supporting evidence.
Mark the statement *TNR* if it is true but the reason given does not support it.
Mark the statement *NT* if it is not true.

Illustrations: 1. ( ) One sees his image in a plane mirror because the image is real.
2. ( ) The medium governs the velocity of sound for we find the velocity increasing with temperature.

Type II: Mark the statement *S* if it holds for sound.
Mark the statement *L* if it holds for light.
Mark the statement *B* if it holds for both.
Mark the statement *N* if it holds for neither.

Illustrations: 1. ( ) Velocity less in air than in water.
2. ( ) Shows the Doppler effect.

Although still in the formulative stages, experience so far points up favorably. The student reaction is exceptionally good. It is felt that these are good testing devices as well as good instructional tools.

5. Oblique coordinates in general physics. HENRY S. C. CHEN. *Drexel Institute of Technology*.—In solving problems on statics and motion in general physics, rectangular coordinates are exclusively used. This paper hopes to show that oblique coordinates can be easily used and understood by the first-year students. Examples are worked out, which show cases where oblique coordinates seem to be the more natural system to use.

6. Avenues of cooperation between physics and the engineering college. G. P. BREWINGTON, *Lawrence Institute of Technology*.—The place of the conventional freshman or sophomore physics course in the engineering college has recently been questioned on the ground that what is taught is largely engineering and also that almost no conception of modern physics can justifiably be conveyed in the time available. Several suggestions will be made by means of which the physics course will be more acceptable to the engineering college. Physics departments can also serve the engineering college by offering certain advanced modern physics courses, mathematically somewhat beyond most current texts. Physics departments should, therefore, make a study of our product (students) in terms of the needs of our best customer (the engineering college).

7. Scientific womanpower—our country's need and what women's colleges are doing to supply physicists. SISTER MARY THERESE, *Mundelein College*.—In the February, 1951, publication of *Bulletin of the Atomic Scientists*, Dr. Eugene Rabinowitch discussed the necessity in this country for augmenting our scientific personnel if we are not to run the danger of being outnumbered in scientists and technicians by the nations of Europe, in particular, the Soviet Union. "There is in this country one—and only one—large reservoir of potential scientific, medical, and engineering 'manpower', which is as yet almost entirely untapped: the American women. . . . Leaders in American education—particularly women's education—and officials responsible for the planning of long-range mobilization of American manpower would be guilty of a fateful omission if they did not consider, at the present time, the necessity of bringing about a radical change in the accustomed American attitude toward the study of science, medicine, and engineering by women. The time to direct thousands of girls who graduate from high schools into these fields of study, is *now*."

At about the time that this article appeared questionnaires were being returned to the Physics Department of Mundelein College which was studying the status of physics in the liberal-arts colleges for women. The present paper discusses the efforts which should be made by physics departments in women's colleges throughout the country and how well the colleges are contributing to the supply of scientific womanpower in the field of physics.

8. A program to inform high school students about opportunities in physics. W. C. KELLY, *University of Pittsburgh*.—To meet the increasing need for physicists, college teachers, and high school teachers must work in close cooperation to inform able students of opportunities in this field and to encourage them to major in physics in college. For the past three years, the University of Pittsburgh has sponsored physics lectures and career conferences for high school students. Each year, an invited speaker known for his ability to keep young people interested gives a physics lecture, usually with demonstrations, to an audience made up of students and teachers from Western Pennsylvania. After the lecture, the students first attend a conference at which they are told about physics as a career and then visit the physics laboratories. More than a thousand students have come to these annual lectures. In addition, our physics department has given a popular series of demonstration lectures for high school students during the school year which review mechanics, light, etc., after the students have encountered them in their high school course. The department works with various scholarship-granting agencies and has been able to get scholarships for students of high scholastic ability who want to major in physics. The paper evaluated what had been accomplished by the program and suggested some ways by which the AAPT could help such work.

9. Undergraduate student participation in academic research. STANLEY S. BALLARD, *Tufts College*.—Government-sponsored research now tends to dominate physics

departments, not only in the larger universities, but also on small and medium-sized campuses. It must not be forgotten, however, that academic research grants may still be sought; grants of the order of $500 to $1000 are now relatively easy to obtain in physics. These present a most effective vehicle for research in fundamental physics, whereas sponsored research tends in the direction of applied physics. The scope of such academic-type projects is necessarily modest, but the results obtained should nevertheless be significant. For the undergraduate physics major who participates, the small academic project may prove to be more valuable than the larger government project; in the latter he may be "lost in the shuffle" because of his relative inexperience. It is felt that semi-independent work under such a grant offers some advantages over the "senior problem" plan used so successfully in certain colleges. Participation in research projects by undergraduates may qualify them for associate membership in Sigma Xi and for certain prizes and scholarships awarded in the senior year. Also, this experience will certainly be helpful to them in their subsequent graduate work, and it should assist them in obtaining graduate fellowships. A four-year period of experience with student participation in academic research in the physics department of Tufts College was reviewed as an illustration in point.

10. The use of the group experiment in preliminary stages of laboratory instruction in elementary physics. W. P. RANEY and R. A. PECK, JR., *Brown University.*—What is believed to be a widespread condition has been noted at Brown University: Students in the laboratory sections of the elementary physics course suffer from general inability to grasp the import of conventional laboratory instructions and to appreciate the bases of standard experimental procedures and equipment. In an attempt to better this condition a group-experiment approach for the first sessions has been developed, producing immediate student participation and considerable original thought about aims and methods in the case of three simple investigations. The instructor acts as moderator, and the students are led essentially to devise and criticize their own experimental procedures as a group and thus to improve them.

Details were presented about the three experiments used at Brown University: Motion of a freely falling body, two-dimensional motion of a body under constant acceleration, and the ballistic pendulum.

11. The use of mnemonics in physics teaching. W. E. HAISLEY, *Brown University* (By title).—In a recent group of letters[1] submitting mnemonic devices for Maxwell's thermodynamic equations, a sharp difference of opinion was apparent as to the propriety of using, to facilitate the recall of these relationships, any devices except "natural" ones arising out of their derivation.

Opposition to mnemonic devices probably arises out of the feeling that memorization is apt to degenerate into a short circuit for genuine grasp of underlying principles. Trick mnemonics are suggestive of "formula-plugging," and the idea that their use is in some way immoral is not surprising. Admitting, however, that memory is no substitute for understanding, there are certainly many practical situations (use of the multiplication table, for instance) where grasp of underlying principles cannot replace immediate and spontaneous recall. There is no clear-cut evidence that a trick mnemonic may not contribute in a positive way, by freeing the mind to concentrate on basic principles. It is suggested that perhaps, after all, a mnemonic should be judged by its simplicity and effectiveness and not by its artificality.

[1] C. M. Focken, Am. J. Phys. **16**, 450 (1948), and others listed by F. H. Crawford, Am. J. Phys. **17**, 450 (1949).

12. Book and sound records of scientific meetings. J. G. WINANS, *University of Wisconsin.*—Tape recording of nearly every address at the June, 1952 meetings at Iowa City provided a library of records and a set of manuscripts. The manuscripts were edited and duplicated by the speakers. Assembly of the duplicates provided 250 copies of a 475-page book record of the meetings. The cost was about $2.00 per volume for typing, mimeographing, assembling, binding, and mailing. A good plan for the future may be to have speakers send stencils ready to run. Possible benefits of such records as these are (1) shock to speakers in reading and hearing what they actually said, (2) preservation of reports of special interest, (3) preservation of the voices of scientists for future generations, (4) provision of a reference book for the study of things only partially understood or of reports of particular interest, (5) discussion of controversial reports by mail, (6) preparation of better articles for later publication in an international journal, (7) provision of an opportunity to read reports not heard because of duplicate sessions, (8) production of some physics literature in noncondensed and readable form, (9) provision of the information needed for reporting the meetings to scientists unable to attend.

13. Installation and operation of ten-kilocurie cobalt-60 gamma-radiation source. JOHN V. NEHEMIAS, L. E. BROWNELL, W. W. MEINKE, and E. W. COLEMAN, *Fission Products Laboratory, University of Michigan.*—A discussion was presented of the problems involved in design and construction of housing for a ten thousand curie gamma-source suitable for maximum-dose, maximum-volume irradiation of biological or chemical samples. The actual installation and operation of the source is discussed, with reference to hazards involved and safety precautions taken. A short report was presented of experimental results to date.

14. Intermediate laboratory physics—methods and aims at the University of Rhode Island. WILLIAM E. PLAISTED, *University of Rhode Island.*—A brief description of the typical student's previous preparation and of courses concurrent with the intermediate laboratory were given. The possibility of close coordination between experiments, these courses, and the laboratory was shown by several illustration experiments. In conclusion the aims of the laboratory were shown to be a development of individuality in the student such that he would be able to complete a project of his own choice.

15. Intermediate laboratory experiments at Cornell University. B. D. McDaniel, *Cornell University.*—A few general remarks were made concerning the nature of the Cornell University teaching laboratories, emphasizing the economy and flexibility arising from the unification of the junior, senior, graduate, and advanced laboratories. The nature of the experiments was typified by a recently developed laboratory experiment in mechanics. This experiment utilizes a torsion pendulum in the study of mechanical forced oscillations. The torsion pendulum, resonant at about 5 cps, is driven by an accurately controlled gear system (± 0.1 percent in frequency). The torsion disk is damped by an electromagnet. Phase and amplitude of the oscillations may be studied as a function of frequency and damping. Exponential decay as well as beats between transient and steady-state solutions may be observed. Four methods of determination of Q and three methods of determining the resonant frequency are described. The residual damping yields a Q as great as 600.

16. A course in laboratory and shop processes for physics majors. Walter Eppenstein, *Rensselaer Polytechnic Institute.*—A three-semester course in laboratory and shop processes is given to all physics majors. The first two semesters are offered during the sophomore year and cover a number of experimental techniques which are usually not encountered in the regular physics laboratories. The students do bench and machine shop work on instrument parts, soft and hard soldering, and glass blowing. They work on high vacuum systems, metal sputtering, and evaporation, in this way becoming familiar with different available materials and the names of the manufacturers of these materials. The students are also introduced to experimental methods which encourage original thinking, thus preparing them for their more advanced laboratory work. Instead of a final examination the students give talks on various experimental techniques such as are used in physics. These talks usually lead to a class discussion on the subject.

The third semester of the course is given during the senior year and consists of the construction and testing of an electronic piece of apparatus, such as an audio amplifier.

Frequently a number of graduate students take one or more of these courses, and find them good preparation for their thesis work.

17. Selected experiments for a course in advanced optics. Shang Yi Ch'en, *University of Oregon.* (Paper presented by Will V. Norris).—Ten laboratory experiments in geometrical optics and eighteen in physical optics were chosen for a course in advanced optics, with three lectures and one laboratory per week for three terms. The laboratory experiments are arranged to match very closely with the lectures. About eight experiments are given with the emphasis that the students must follow closely the laboratory instructions so that they learn the technique of handling the instrument. About twelve experiments are given so that the students can verify what they have learned from their books or from the lecture; and eight experiments are assigned so that the students, with the techniques they have learned, can use their own wisdom or judgment in carrying out the assigned measurement. The laboratory report contains no description of procedures if they can be found in any books. In addition to the calculation of data, discussion of results, or suggestions for improvements, a few questions are usually supplemented for them to answer.

18. Cardinal points of a compound lens: an advanced laboratory experiment. Thomas B. Brown, *George Washington University.*—The lens is mounted on an optical-bench carriage and its two principal focal points, F' and F'', are located by the autocollimation method. A conjugate pair of extra-focal distances, x' and x'', then are obtained by focusing on a ground-glass screen an image of an illuminated aperture. The distance from F' to the aperture is x' and x'' is the distance of the image from F''. The focal length is then computed by Newton's formula, $f=(x'x'')^{\frac{1}{2}}$, and with it the nodal points are located relative to F' and F'' in the usual manner. Precise results require that certain precautions be taken in the experiment, and these were described. It is desirable that the lens be a combination of two or three thin lenses, so that the results may be checked by computation from the focal lengths and relative positions of the component lenses. Results are as good as those obtainable with the better-known nodal-slide method. A green fluorescent lamp provides a bright, uniformly diffused source of light for lens experiments, and it may be considered almost monochromatic for this purpose.

19. The tapered-film interference filter. Winthrop R. Wright, *Swarthmore College.*—The numerical aspects of thin film interference are directly shown by the small interference filter of tapering thickness which is available from Bausch & Lomb. Two orders of 4340 occur in the range of the wedge with intervening transmission maxima for 5460 and 5780. The filter is laid on a ground glass, illuminated by a mercury arc, and the positions of the four maxima are marked on the glass. The relative positions are plotted against wavelength. The resulting straight line exhibits the uniformity of the taper and serves to set the order of interference.

The filter, with means for rotation, is mounted on an optical bench. The region which gives maximum transmission for 5780 at normal incidence is set on the axis and the wedge is then rotated to the position of transmission for 5460, observation being made with a small telescope focused on the axial region. The angle of rotation (*ca.* 30°) is compared with the angle in the film ($\cos^{-1}$ 5460/5780) to obtain the refractive index. There is observable birefringence at oblique incidence.

20. Projects in advanced electricity for undergraduate students. Chas. Williamson, *Carnegie Institute of Technology.*—Our lecture courses in electricity and magnetism and in electronics, both for physics majors, are accompanied by laboratory courses in which the

students perform some routine experiments in order to acquire a backlog of technique and understanding, and then elect one or more projects in which they plan and perform their own work with minimum help from the instructor. Both verbal and written reports are called for. Projects are of three kinds; in the first, only intelligent selection and use of printed references are required; in the second, construction and testing of standard laboratory devices are undertaken; in the third, new designs are taken from current literature and reproduced with modifications originated by the students themselves. Lists of typical projects were given in this paper.

21. Some quantitative experiments with the magnetic top. FRANK VERBRUGGE, *Carleton College*.—An air-driven magnetic top rotating with a uniform angular speed precesses with a frequency determined by the combined torque of its own weight and of an external magnetic field, produced by a pair of Helmholtz coils. A series of measurements of precessional frequency and angular momentum with various currents determines the center of mass for the top and the product HM for the spinning dipole. This product agrees with that obtained when the dipole is set in oscillation in the magnetic field and is shown to be proportional to the current. If an independent measurement of H is made, the magnetic moment M of the dipole and its gyromagnetic ratio (in this instance, a variable quantity) can be determined.

22. Experiment on Maxwellian velocity distribution of electrons. ROBERT R. MEIJER, *George Washington University*.—The experiment suggested is a simplified modification of one described by Harnwell and Livingood[1] on electron velocity distribution. The special tube-and-heater circuit they describe is replaced by a commercial tube having an equipotential cathode such as a 76 (with the grid connected to the plate) or a 6H6 and a filament transformer (or a 6-volt storage battery). If cathode temperatures are to be determined and checked against independent measurements, the G. E. FP-400 kenotron diode could be used. We are now trying this tube. The FP-400 can also be used for experiments on (1) Richardson's law for thermionic emission and (2) the determination of e/m by the magnetron method.[2]

[1] Harnwell and Livingood, *Experimental Atomic Physics* (McGraw-Hill Book Company, Inc., New York, 1933), p. 201 *ff*.
[2] D. H. Tomboulian and P. L. Hartman, Am. J. Phys. **7**, 403 (1939).

23. A laboratory experiment on the dielectric constant of gases. FRANCIS T. WORRELL, *Rensselaer Polytechnic Institute*.—An experiment is described in which the dielectric constant of a gas is measured by a beat-frequency method. By the use of this method the difference between the dielectric constant of a gas and that of vacuum, rather than the ratio of the two, is measured directly. It is possible with reasonable care in measuring the difference to get excellent precision in the absolute value. The method was analyzed, and sources of errors were discussed.

24. An undergraduate electron diffraction experiment. WILLIAM C. ELMORE, *Swarthmore College*.—A simplified version of Thomson's original electron diffraction apparatus has been constructed for an undergraduate laboratory in modern physics. Electrons from a gas discharge running at 18 kv and 3 ma are collimated by 0.01-cm diameter holes in two thin Mo disks separated by 10 cm, one disk being set in the aluminum anode. Collimated electrons pass through a student-made specimen consisting of silver evaporated to semitransparency on a thin film of celluloid, and continue 30 cm to form diffraction rings on an 8-cm diameter fluorescent screen. The screen is coated with No. 1 phosphor from a surplus C-R tube, and concentric circles in 2-mm increments are scribed in the coating to permit measurement of ring diameters without parallax error. The screen is made conducting by a thin layer of evaporated aluminum. The diffraction tube connects to an all-metal, oil-diffusion pumping system, also used with bell jar for preparing specimens. Four or five diffraction rings are observed directly in a darkened room. The measured diameters of the rings, the discharge-tube voltage, and the known crystal structure of silver are used with the de Broglie wavelength formula to evaluate Planck's constant, which usually agrees with the accepted value to about one percent.

25. A simple apparatus for central force investigations in an advanced undergraduate laboratory. STEPHEN BERKO, *University of Virginia* (introduced by J. W. Beams).—The technique of using simulated potential fields by means of rubber surfaces and the experimental study of the motion of particles in such fields is widely known.[1] The apparatus to be described has been designed to study by simple means the central force problem and simulate Rutherford scattering. It consisted of a circular framework holding an elastic rubber sheet, the surface of which had essentially a constant/R form, thereby simulating attractive or repulsive Coulomb potentials.

Small steel balls were given known initial kinetic energy by a slide and were rolled onto the rubber surface. Stroboscopic pictures of the motion of the balls were taken, the light being supplied by a type 678-A, G. R. stroboscope. Displacement, velocity, and acceleration as a function of time could be read directly from the photographs. The repulsive potential yields exact hyperbolas and Rutherford's formula relating impact parameter to scattering angle can be checked. Hyperbolas and precessing ellipses for attractive surfaces allow the study of conservation of angular momentum and areal velocity. The experiment also provides a means of introducing the student to photographic and darkroom techniques; slides taken by the class will be shown.

[1] A previous application for demonstration was pointed out to the author just prior to submission of this abstract [C. L. Henshaw, Am. J. Phys. **11**, 47 (1943)].

26. A demonstration of the Doppler effect. ROBERT H. RANDALL, *The City College of New York*.—Most of the usual lecture-room methods of demonstrating the Doppler effect for sound involve, unavoidably, interference principles as well. In some cases the interference effects due to the combination of the direct and certain reflected waves completely mask any sensation of pitch change.

In the present method, a 6-inch loudspeaker serving as a sound source is mounted on an arm free to revolve in the horizontal plane about a vertical axis. A commutator on the shaft connects the loudspeaker to an electrical oscillator twice during a revolution, once when the speaker is approaching the hearer and once when it is receding. Thus, two short duration sounds, separated by silence, are available for comparison as to pitch. In addition, the arrangement is such that through diffraction principles a broad cone of sound is projected in each case in the direction of the hearer only. In this way the important complications due to reflection at the front of the room are avoided. An optimum sensation of pitch change is secured with a frequency of about 3000 cycles sec^{-1}, as predicted from data on hearing properties.

27. An experimental encounter with bifilar pendula. RICHARD M. SUTTON, *Haverford College.*—A simple, inexpensive experiment adaptable to either elementary or advanced laboratory is described, the aim being similar to that described for ring pendula in "Empirical Equations" by V. E. Eaton.[1]

A uniform dowel rod is hung from a rigid horizontal beam by two vertical threads of equal length attached to its ends. It is set into oscillation about a vertical axis through its center of mass; or it may also be made to oscillate at right angles to its length, about an axis coinciding with the supporting beam. How does the period in each case depend upon the length L of the supporting threads? Which of these two modes of oscillation is the quicker? How is the period affected by change in mass M and in length of rod D? Answers to these questions may bring certain surprises to the experimenter. First, it is not surprising that rods of equal length D but of different mass M supported by cords of the same L have equal periods: the period does not depend upon the density of the rod. But one might expect the period to depend upon the length of rod D as well as on the length of supporting cords L. This is *not* so: a long rigid rail will swing in the same period as a lead pencil when each is supported by cords of the same length. If one were asked to decide in advance which of the two modes of oscillation would occur in the shorter time, for a given pendulum, one might suppose (as the author did) that the moment of inertia of the rod would play such an important part in the first type of oscillation that the second type of swing would be quicker. This is *not* so: the rotary oscillation is shorter by a constant factor of $\sqrt{3}$, a factor which crops up immediately in the experiment. Analysis of the mechanics of the two swings shows that this factor is warranted.

A set of bifilar pendula, mounted as described, provides the means for obtaining data for two empirical equations. Several advantages are immediately apparent: first, they are very inexpensive; second, they may be readily installed, for one may in a few minutes mount a series of rods of different length D, supported by threads of different length L. Several rods may be suspended from a single supporting beam. Third, the range of lengths readily installed surpasses the range of diameters available for ring pendula, as rings more than 40 cm in diameter are hard to obtain. Fourth, almost complete lack of experience with such pendula presents the student with an empirical situation he has never had occasion to observe and think about before. The surprises are intriguing: the best students will soon contact the instructor for an explanation of that $\sqrt{3}$ factor.

By using both modes of oscillation, the student may rapidly obtain two sets of observations which, when graphed on log-log paper, give parallel lines (same power of the length L) but with different intercepts. The student may then see how well the experimentally determined constants obtained from his graph agree with $2\pi(L/g)^{\frac{1}{2}}$ and $2\pi(L/3g)^{\frac{1}{2}}$, the values indicated by theory with which he may not initially be expected to have familiarity. As a matter of experimental "neatness," one may arrange successive pendula in a series so as to have lengths L which increase in a geometric series, thus giving data points equally spaced on the log-log graph.

And for the student whose talent needs to be stretched, let him compute where to attach the threads to a dowel rod so that the two modes of oscillation of the rod as a pendulum shall be the same!

[1] From *Selective Experiments in Physics* (Central Scientific Company, Chicago).

28. Demonstration experiments. ERIC M. ROGERS, *Princeton University.*—(1) Use of a simple calibrated spring to weigh small objects and measure small forces, to avoid using almost invisible weights in large lectures. As an example, the spring was used to measure (a) the repulsion between two charged balls; (b) the force on one ball between parallel plates with known potential difference. Measurement (a) gives a rough test of Coulomb's Law, and (a) and (b) combined give the constant in Coulomb's Law.

(2) Experiment to show that surface tension is independent of area or thickness of soap film. The usual "window-shade" demonstration with a movable slider pulling a plane soap film is done with a second slider, at the other end of the film, attached to a very light spiral spring. Stretching the film does not change the spring.

(3) The "weight" of a man in an accelerating elevator. A huge Atwood machine made with a bicycle wheel carries a spring weighing scale to measure the "weight" of one load. The scale shows that the load is accelerating down whether it is moving up or down or is momentarily at rest.

29. Presenting virtual objects by demonstration and a graph to present real objects and images. MARVIN J. PRYOR, *New York State College for Teachers, Albany.*—(1) The demonstration equipment consists of a pair of lenses mounted on hinged boards. Light from a lamp filament forms bright images on each of two screens which may be observed by a class. The second lens is a half-lens, being cut at a diameter so that the light forming one image has passed over the half-lens, while light forming the other image has passed through the half-lens. Turning one board at the hinge leaves only the half-lens and the two screens. Lines drawn on this board show to the class that the distant screen is at the position of the virtual image of rulings on the nearer screen. It is made quite evident that for the half-lens the positions of real object and virtual image become the positions of real image and virtual

object when the lamp and whole lens are returned to position. Companion boards are used to show these virtual objects. One uses a positive half-lens, the other a negative half-lens.

(2) A large graph is arranged so that a single lamp filament at the origin serves as object for identical positive lenses placed along the p axis. Images are formed on screens placed along a hyperbola drawn on the graph. The relations of both size and distance of image to object distance are clearly demonstrated.

30. A physical-optics approach to lens and mirror problems. LEWIS S. COMBES, *Tufts College*.—By introducing the concept of the *vergence* of a light wave, lens and mirror problems can be solved by a physical rather than a geometrical approach. The vergence of a wave front at any point in a medium is defined as the refractive index of the medium divided by the radius of curvature of the wave front. Converging light has a positive vergence and diverging light a negative vergence; the vergence of plane waves is zero.

The idea of vergence may be introduced in an optics class as soon as Snell's law has been derived. When light passes through a plane surface separating two optical media its vergence does not change, although in general the curvature of its wave front does change. Whenever refraction or reflection occurs at a curved surface, the vergence of the light changes, and it can be shown that the optical strength (power) of a spherical surface is equal to the change in vergence produced by that surface. Then a very simple relation holds in all cases, namely, the vergence of the refracted (or reflected) light V is equal to the vergence of the incident light U plus the optical strength of the refracting (or reflecting) surface S, i.e., $V=U+S$. This easily remembered relation may be applied successively to any number of refracting and/or reflecting surfaces. In this way any lens or mirror problem involving only paraxial rays can be solved.

31. The Steadman convergence figure of light intensity. F. M. STEADMAN, *Los Angeles 5, California* (Read by title only).—Any unit of matter, from the individual surface molecule up to each larger letter on this page, is the natural point of a converging light pencil, whose natural base is the whole area of the light source which is illuminating it. A diagram has been prepared to represent the Steadman convergent figure of light intensity, based on the $f/64$ cone-form of solid angle as a simple unit of the hemisphere. This unit has been named by me the *stead*.

32. The concept of radiation measurements. DONALD J. LOVELL, *Westinghouse Electric Corporation, Friendship Airport*.—Concepts are defined which are suitable for discussing all portions of the electromagnetic spectrum. The concept of "light" is limited to that portion of the spectrum called the visible region. The term radiation is employed as a more general concept. Similarly general and fundamental terms are used to illustrate techniques of teaching radiation without the confusion frequently connected with the psychophysical concepts of photometry.

33. On the history of electricity prior to 1600. DUANE ROLLER, *Wabash College*, and DUANE H. D. ROLLER, *Harvard University*.—Some of the important aspects of the history of electricity before 1600 are described. The known phenomena and the roles of explanation and of hypotheses are discussed.

34. Interpretation of the Ampère experiments. DOMINA EBERLE SPENCER, *University of Connecticut*.—The Ampère experiments[1] on forces between current-carrying wires are examined to determine all possible interpretations. From the infinite number of possible expressions for force between current-carrying elements, which satisfy the Ampère experiments, it is shown that only five can be derived from an expression for force between moving charges and these five do not include the equation ordinarily called[2] "Ampère's equation" in physics textbooks. Some implications of this peculiarity, which have a bearing on the teaching of the electromagnetism, are pointed out.

[1] A.-M. Ampère, Mém. acad. sci. Toulouse 6, p. 175; *Collection de Mémoires relatifs à la physique* (Paris, 1887), Vol. 3, p. 1.
[2] M. Mason and W. Weaver, *The Electromagnetic Field* (University of Chicago Press, Chicago, 1929), p. 184; N. H. Frank, *Introduction to Electricity and Optics* (McGraw-Hill Book Company, Inc., New York, 1940), p. 95.

35. Electroluminescence of phosphors. ALFRED E. MARTIN, *Sylvania Electric Products Inc., Bayside, New York*.—Certain aspects of the general phenomenon, luminescence, have been familiar for a matter of three centuries. Electroluminescence of phosphors, however, has been recognized only rather recently. Luminescence is the process whereby matter, having previously absorbed a quantity of incident radiation, subsequently emits a portion of the absorbed energy as nonthermal, quantum radiation. Electroluminescence is that aspect of luminescence in which there is a direct transfer of energy from a varying electric field to a suitable phosphor embedded in a dielectric material. The accompanying visible emission spectrum from the phosphor is described as electroluminescent radiation.

The present paper describes a new type of light source resulting from basic researches into this phenomenon. It is shown that a true area source of illumination has been developed, in contrast to the point and line sources now predominantly in use. Descriptive quantitative data are also presented in order to illustrate the effect on electroluminescence emission of such parameters as applied voltage, supply frequency, dielectric hardness, power input, and temperature. Typical electroluminescence plaques were demonstrated. Studies of the fundamental properties of the phosphors used in electroluminescence plaques are under way. Some valuable information has been gained by examination of the optical absorption and emission spectra of these phosphors.

36. Demonstration of the velocity distribution in laminar flow. ALEXANDER KOLIN, *University of Chicago*.—The method of visualizing the local fluid velocity is illustrated on the example of laminar flow through a straight pipe of circular cross section. A wire is inserted into the pipe so as to run along a diameter. It is connected to the positive pole of a 110-volt dc source. A second wire, placed about

an inch upstream, serves as the cathode. The fluid consists of a mixture of water, glycerine, starch, and potassium iodide. Iodine which is liberated at the anode wire reacts with starch. A deep blue cloud is formed about the wire. This cloud is deformed into a parabola when the fluid is set in motion.

Report of the Committee on Awards

At the Annual Meeting of the American Association of Physics Teachers in New York in 1952, the Council voted to extend the duties of the Committee on Awards. In addition to the responsibility of choosing the Oersted Medalist, the Committee was told to submit recommendations to the Council for Honorary Membership and for Citations. The former was to be granted to men who have furthered the cause of physics teaching but who would not be expected to be members of our Association, and the latter to members who have given particularly distinguished service to the ideals and purposes of the Association.

The Committee met in Chicago in October, 1952 and at Iowa City in June, 1952 and engaged in several mail ballots. The result of these considerations is contained in the following recommendations which were moved and carried unanimously at the Meeting of the Council at Cambridge, Massachusetts, on January 22nd and also at the Business Meeting of the members on January 24th, 1953.

Honorary Membership

The Committee on Awards recommends to the Executive Council that honorary membership be conferred upon the following men:

(1) R. POHL, Professor of Physics at the University of Göttingen, whose textbooks on physics combine the rigor of fundamental physics with extraordinary originality of exposition and illustration, and whose lecture demonstrations are famous throughout the world.

(2) J. H. KEENAN, Professor of Mechanical Engineering at the Massachusetts Institute of Technology, whose books on thermodynamics, properties of steam, and similar subjects, are unusually well grounded in physics, and whose articles on engineering and on engineering education have always emphasized the importance of the study of physics.

Citation

The Committee on Awards recommends to the Executive Council that citations be awarded to the following men:

(1) HAROLD K. HUGHES, for his painstaking work as chairman of the Committee on Letter Symbols, which culminated in the publication of a standard list of physical symbols that has been adopted as American Standard Z10.6 (1948) and has exerted considerable influence on authors of textbooks and periodical articles.

(2) E. C. KEMBLE, for his brilliant work as chairman of the Coulomb Law Committee, leading to an important contribution to the subject embodied in a comprehensive report published in the *American Journal of Physics*.

(3) THOMAS H. OSGOOD, for his splendid record as Editor of the *American Journal of Physics*. Under his guidance the *Journal* has retained its place as one of the most readable and attractive mediums in the field of cultural and pedagogic physics.

(4) R. R. PALMER, for his loyalty and efficiency in assuming the duties of program chairman of the Association for the Summer Meeting at Iowa City in 1952 and the Annual Meeting at Cambridge in 1953.

(5) K. LARK-HOROVITZ, for his activities in the training of teachers of secondary school physics in Indiana, his faithful and highly effective work as chairman of the Cooperative Committee on the Teaching of Science and Mathematics, and his outstanding ability to combine the duties of a teacher with the responsibilities of a research physicist.

(6) M. W. WHITE, for his faithful and successful efforts, as chairman of the Membership Committee, to increase the roster of the Association and to bring to this society credit and prestige.

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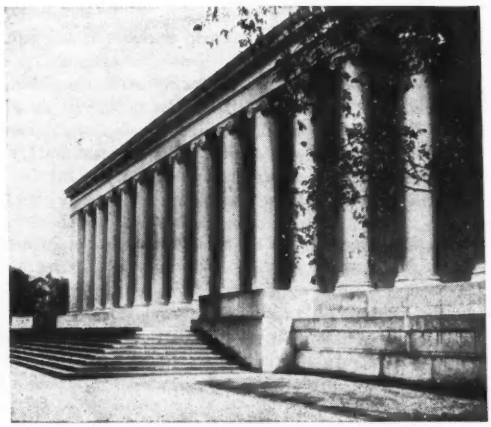

Mellon Institute

The Mellon Institute, 4400 Fifth Avenue, in the Civic Center of Pittsburgh, is the location of the summer meeting of the American Association of Physics Teachers on June 25–27, 1953. An air-conditioned auditorium has been reserved for the sessions of the Association.

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CONTENTS

AMERICAN JOURNAL OF PHYSICS VOLUME 21, NUMBER 5 MAY, 1953

Dimensional Theory: Dimensionless Secondary Quantities . . . . . L. L. WHYTE 323

Course in Radioactive Determinations . . . ROBERT H. MCFARLAND AND RICHARD E. HEIN 325

Physics of Microwave Calorimeters . . . . . ERNEST C. OKRESS 330

Analysis and Synthesis of Optical Images . . . . . J. ELMER RHODES, JR. 337

Prenatal History of Electrical Science . . . . . DUANE ROLLER AND DUANE H. D. ROLLER 343

Experiment on Cosmic Rays . . . . . P. G. GUEST AND WENDELL M. SIMMONS 357

Trend Analysis of Physical Data . . . . . G. E. HUDSON 362

Presentation of Oersted Award to Professor Richard M. Sutton . . . . MARK W. ZEMANSKY 368

Heritage of a Physics Teacher . . . . . RICHARD M. SUTTON 369

Electronics in a Sophomore Physics Course . . . . . E. ALLAN WILLIAMS 373

Physics of the Organ Flue Pipe . . . . . DERWENT M. A. MERCER 376

Notes and Discussion:

Ideal Gas Equation . . . . . LUIGI Z. POLLARA AND FRANK Z. POLLARA 387

On the Resolving Power of a Prism . . . . . K. MAJUMDAR AND MAHENDRA SINGH SODHA 387

Group-Phase Velocity Demonstrator . . . . . ROBERT KATZ 388

Dissipation Factor Anomaly . . . . . RAYMOND M. BELL AND HARRY HILL 389

Striations in Electromagnetic Stationary Waves . . . . . GRANT O. GALE 389

Letters to the Editor:

Need for More and Better Trained High School Physics Teachers . . . . . DONALD C. MARTIN 390

Observations on the Smoke Trail of a Sky-Writer . . . . . JULIUS SUMNER MILLER 391

Problem in Electrostatics . . . . . VERNON CRAWFORD 391

Concerning the Frequencies Resulting from Distortion . . . . . BEROL L. ROBINSON 391

Exciting a Kundt's Tube with a Siren . . . . . K. A. PARSONS 392

Announcements and News:

Summer Meeting of AAPT 356

Practical Aids for Physics Teachers 386

Book Reviews 392

New Members of the Association 399

Mellon Institute 410

Recent Meetings:

Chicago Section 400

Western Pennsylvania Section 400

Proceedings of the American Association of Physics Teachers, Twenty-Second Annual Meeting, January 22–24, 1953 401